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Variability estimation of failure of shallow foundation on clayey soils with a modified Cam Clay yield criterion and Stochastic FEM

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Presentation contents

- Introduction (Scope-goal)
- Presentation of the theory (Porous media, material constitutive model, stochastic finite element analysis)
- Numerical tests on stochastic consolidation with random linear and non linear material properties
- Conclusions

Introduction

- Porous consolidation of clays. ➔ Equillibrium equations of solid coupled with equations of fluid flow

- For lower frequency excitations ➔ u-p formulation which requires less computational effort

- The soil behavior is described with the compressibility factor κ, the critical state line inclination of the soil c and the permeability k. In this context, Modified Cam Clay models are implemented.

Introduction

- In the stochastic finite element method there are two ways of providing the spatial distribution of the input stochastic variables

1. The nodal points are considered as random variables and deterministic shape functions are used for providing the material spatial distribution
2. The spectral representation or the Karhunen Loeve sum, can be applied for providing a random field of the variable under consideration.

Introduction

- In this work the material yield model proposed by Kavvadas and Amorosi is adopted → accurate and reliable material model
- Valid quantitative results for predicting the response for porous consolidation of clays under uncertainty
- The goal is to quantify the uncertainty of the output displacement in relation to the variability of the input, the spatial distribution of the material variables and the eccentricity of the footing settlement.

Dynamic soil-pore-fluid interaction-The Biot problem

- Equation of equilibrium for the soil-fluid mixture

$$\boldsymbol{S}^{T}\boldsymbol{\sigma} - \rho\ddot{\boldsymbol{u}} - \rho_f(\dot{\boldsymbol{w}} + \boldsymbol{w}\nabla^{T}\boldsymbol{w}) + \rho\boldsymbol{b} = \boldsymbol{0}$$

- Equation of balance of the fluid

$$-\nabla p - \boldsymbol{R} - \rho_f\ddot{\boldsymbol{u}} - \frac{\rho_f(\dot{\boldsymbol{w}} + \boldsymbol{w}\nabla^{T}\boldsymbol{w})}{n} + \rho_f\boldsymbol{b} = \boldsymbol{0}$$

- Flow conservation equation

$$\nabla^{T}\boldsymbol{w} + \left(1 - \frac{K_T}{K_s}\right)\boldsymbol{I}\dot{\boldsymbol{\varepsilon}} + \dot{p}\left(\frac{n}{K_f} + \frac{1-n}{K_s}\right) + \frac{n\dot{\rho}_f}{\rho_f} + \dot{\boldsymbol{s}} = \boldsymbol{0}$$

- Darcy law ➔ $\boldsymbol{kR} = \boldsymbol{w}$

- Stress-Strain material law and pore pressure stress equation

$$\boldsymbol{d\sigma}' = \boldsymbol{Dd\varepsilon}_{el}\,,\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p$$

- Boundary conditions of known pressures, displacements, water velocity and stresses in specific parts of the boundary.

Dynamic soil-pore-fluid interaction-The u-p problem

- Equation of equilibrium for the soil-fluid mixture

$$\boldsymbol{S}^T\boldsymbol{\sigma} - \rho\ddot{\mathbf{u}} + \rho\boldsymbol{b} = \mathbf{0}$$

- Flow conservation equation

$$\nabla^T\boldsymbol{k}(-\nabla p - \rho_f\ddot{\mathbf{u}} + \rho_f\boldsymbol{b}) + \left(1 - \frac{K_T}{K_s}\right)\boldsymbol{I}\dot{\boldsymbol{\varepsilon}} + \dot{p}\left(\frac{n}{K_f} + \frac{1-n}{K_s}\right) + \dot{\mathbf{s}} = \mathbf{0}$$

- Stress-Strain material law and pore pressure stress equation

$$\boldsymbol{d\sigma}' = \boldsymbol{Dd\varepsilon_{el}}\,, \boldsymbol{\sigma}' = \boldsymbol{\sigma} + p$$

- Boundary conditions of known pressures, displacements, water velocity and stresses in specific parts of the boundary.

- This formulation is valid in low frequency excitations in relation with the eigenfrequency of the soil and in static problems.

Dynamic soil-pore-fluid interaction-FEM simulation

- Equation of discretized u-p formulation

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{Ku} = \boldsymbol{F}$$

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C_1} & \boldsymbol{0} \\ \boldsymbol{Q_c^T} & \boldsymbol{S} \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} \boldsymbol{K_1} & -\boldsymbol{Q_c} \\ \boldsymbol{0} & \boldsymbol{H} \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} \boldsymbol{F_1} \\ \boldsymbol{0} \end{bmatrix}, \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix}$$

Where $\boldsymbol{M_1}$, $\boldsymbol{C_1}$, $\boldsymbol{K_1}$ are the standard mass, damping and stiffness matrices respectively and $\boldsymbol{Q_c}$, $\boldsymbol{H}$, $\boldsymbol{S}$ are the coupling, permeability and saturation matrices respectively and are expressed as

$$\boldsymbol{Q_c} = \int_V \boldsymbol{B^T I N_p}\, dV, \boldsymbol{H} = \int_V (\boldsymbol{\nabla N_p})^T \boldsymbol{k} (\boldsymbol{\nabla N_p})\, dV, \boldsymbol{S} = \int_V \boldsymbol{N_p^T N_P} \left(\frac{n}{K_f} + \frac{1-n}{K_s} \right) dV$$

The material yield stress model

- Bond Strength Envelope (BSE)

$$\frac{1}{c^2}\boldsymbol{s}:\boldsymbol{s} + (p_h - a)^2 - a^2 = 0$$

- Plastic Yield Envelope (PYE)

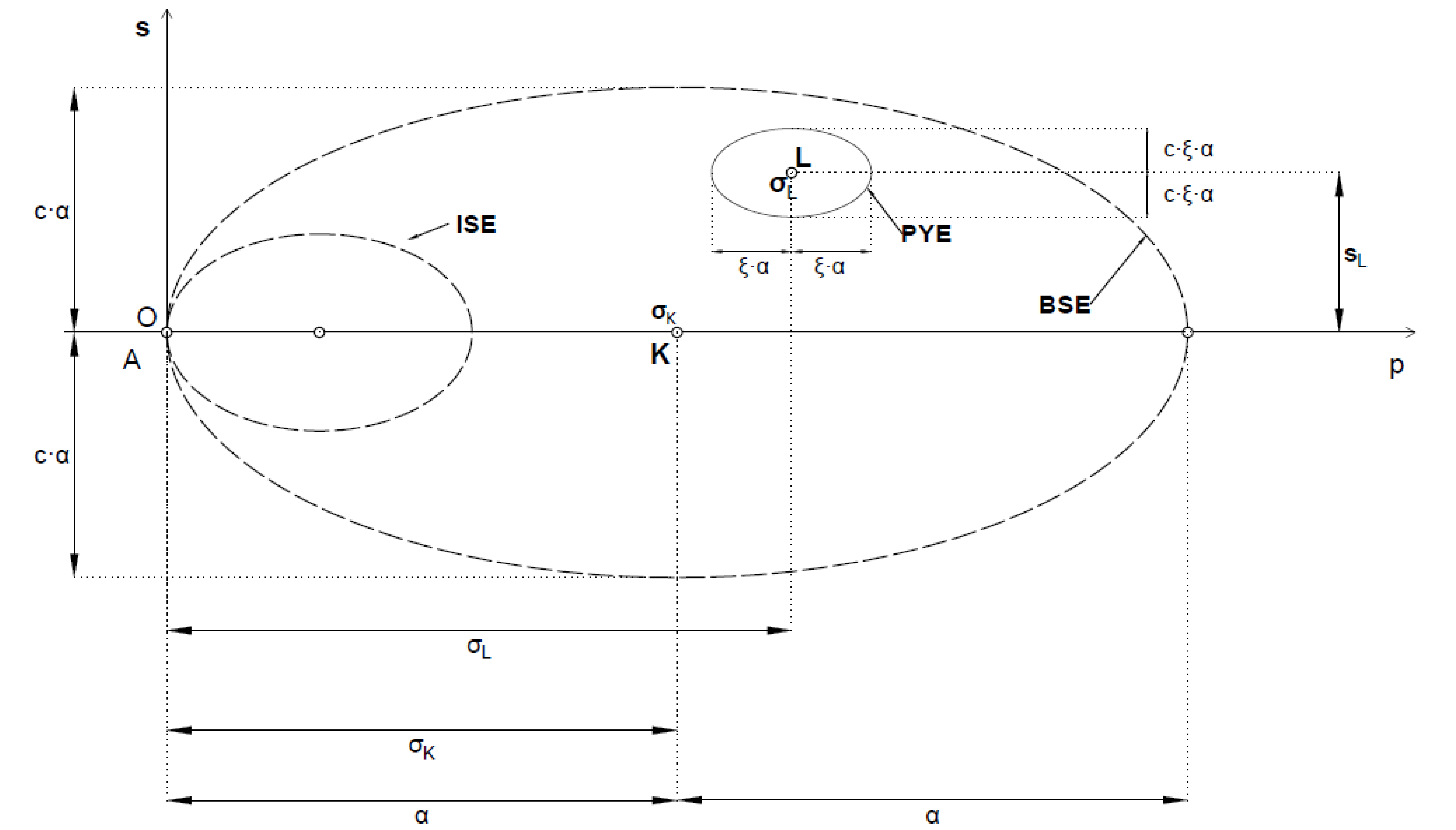
$$\frac{1}{c^2}(\boldsymbol{s} - \boldsymbol{s}_L):(\boldsymbol{s} - \boldsymbol{s}_L) + (p_h - p_L)^2 - (\xi a)^2 = 0$$

- Intrinsic Strength Envelope (ISE)

$$\frac{1}{c^2}\boldsymbol{s}:\boldsymbol{s} + (p_h - a)^2 - a^2 = 0$$

- The PYE is always inside BSE and ISE is the smallest possible BSE

The material yield stress model



Graphical representation of the constitutive model

The Karhunen Loeve Series

- For a random field with zero mean and an autocovariance function

$$C_h = \sigma_d^2 e^{\frac{\Delta \mathrm{X}}{b}}$$

The solution of the integrodifferential Fredholm problem provide analytical expressions for the eigenvalues λ and sinusoidal expressions for eigenfunctions Φ. Therefore a robust equation of the random field and is expressed as follows

$$H(\boldsymbol{x}, \omega) = \mu(\boldsymbol{x}) + \sum_{1}^{M} \sqrt{\lambda_i}\, \Phi_{\mathrm{i}}(\mathbf{x}) \xi_{\mathrm{i}}(\omega)$$

For M number of eigenfunctions chosen to achieve minimum error and $\xi_{\mathrm{i}}(\omega)$ a set of standard normal random variables.

The truncated normal distribution

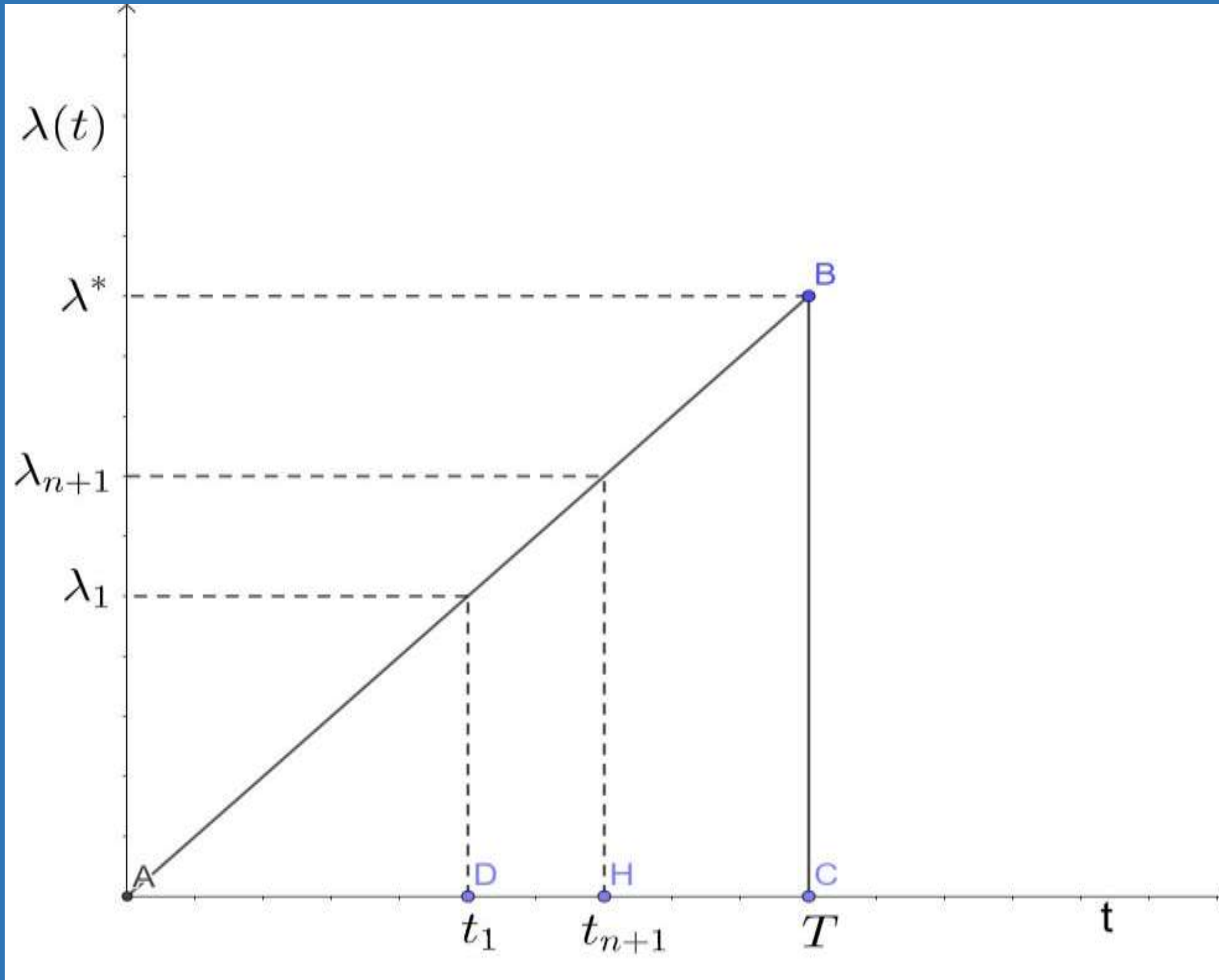
- If the domain of definition of the probability is the [a, b] and we know the normal distribution moments of the PDF in $[-\infty, \infty]$ (μ, σ_d) then the PDF in [a, b] is expressed as

$$g_1(x) = \frac{\varphi(\mathrm{X}_0)}{\sigma_d\left(\Phi(\mathrm{B})-\Phi(\mathrm{A})\right)}$$

Where ϕ and Φ represent the standard normal probability and cumulative distribution function respectively. The X_0 , B, A are the normalized coordinates for x, a, b respectively. The mean value and standard deviation of $g_1(x)$ are functions of the values (μ, σ_d) and the values of ϕ and Φ in A and B

Algorithm for the determination of the failure load in the case of a ramp loading time function

- Let the ramp load function be defined. For determining the factor λ* at the time T a recursive relation is proposed:
- $\lambda_{n+1} = \boldsymbol{a}\frac{\lambda_n t_n}{T} + \mathbf{b}\ \lambda_n$
- The difference between two adjacent iterations at the infinite tends to 0 consequently convergence is achieved if and only if. $\mathbf{0} = \boldsymbol{a} + \mathbf{b} - \mathbf{1}$

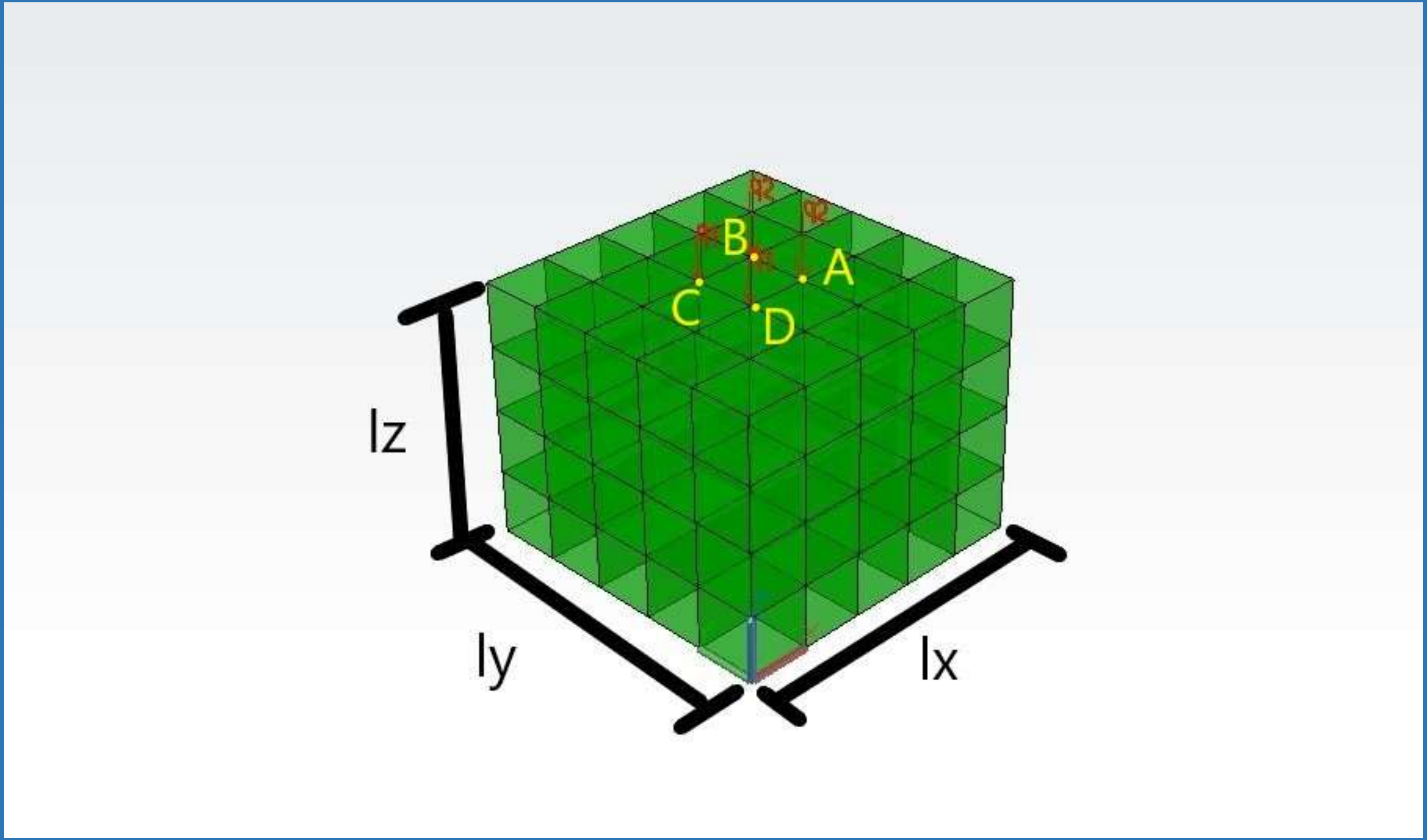


Algorithm for the determination of the failure load in the case of a ramp loading time function

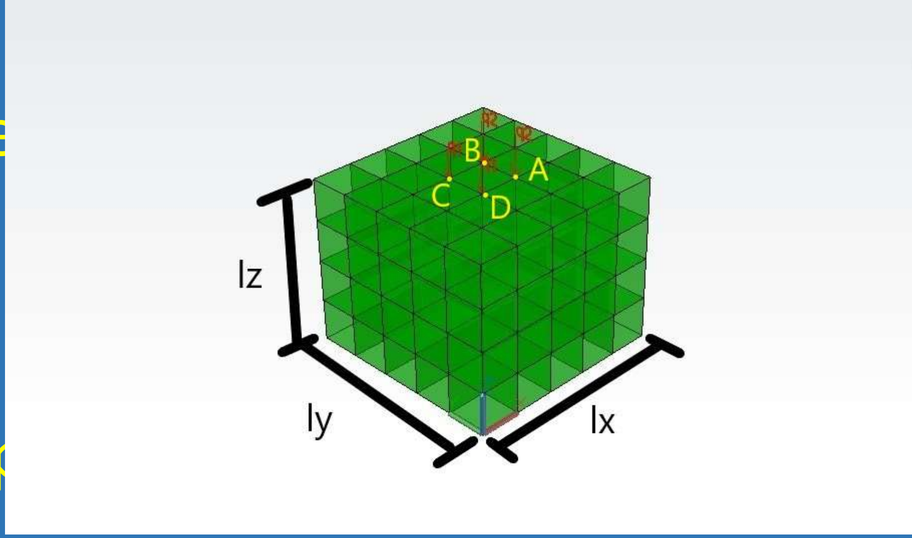
- Only one initial trial guess
- Smaller number of iterations for convergence
- Computational time reduction in comparison with the bisection method in the vicinity of 35%
- Relative error less than 1%

	Bisection Algorithm	Bisection Algorithm	Bisection Algorithm	Proposed Algorithm a=1/3 b=2/3	Proposed Algorithm a=1/2 b=1/2	Proposed Algorithm a=2/3 b=1/3	Proposed Algorithm a=1 b=0	Absolute percentage difference a=1/3 b=2/3	Absolute percentage difference a=1/2 b=1/2	Absolute percentage difference a=2/3 b=1/3	Absolute percentage difference a=1 b=0
Initial value of failure stress (KPa)	5000	5000	5000	5000	5000	5000	5000				
Initial value of safety stress (KPa)	1000	2000	3000	-	-	-	-				
Convergence Tolerance	0.01	0.01	0.01	0.01	0.01	0.01	0.01				
Number of trials for convergence	6	5	5	6	5	4	3				
Displacement of failure at convergence (m)	0.03054	0.03072	0.03054	0.03078	0.03079	0.03071	0.03072	0.78	0.80	0.54	0.58
Load of failure at convergence (KPa)	4484.38	4478.52	4484.38	4555.60	4518.22	4505.94	4477.50	1.59	0.75	0.48	0.15
Computational time (mins)	750	833	663	631	584	475	302	4.83	11.92	28.36	54.45

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description



Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description



- Porous problems and solid problems
- Shallow foundation with linear distributed equivale

Forces for a combination of N,M and 4 eccentricities

e=0, h/12, h/6, h/3

- 8 node Hexa with linear shape functions for u and p
- Initial stresses ➔ Geostatic vertical stresses and

horizontal stresses=100 KPa

- Boundary conditions ➔ **u**(z=h)=**0**
- Deterministic calibration parameters, Poisson ratio, plasticity hardening parameters
- 100 samples of Monte Carlo simulations considering the Latin Hypercube Sampling

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

κ	c	Abbreviation
Constant	Constant	**S**-κ_C-c_C
Linear	Constant	**S**-κ_L-c_C
Constant	Random	**S**-κ_C-c_R
Linear	Random	**S**-κ_L-c_R

Solid analyses performed.

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

κ	c	k	Abbreviation
Constant	Deterministic	Deterministic	**P**-κ_C-c_D-k_D
Linear	Deterministic	Deterministic	**P**-κ_L-c_D-k_D
Constant	Deterministic	Random	**P**-κ_C-c_D-k_R
Linear	Deterministic	Random	**P**-κ_L-c_D-k_R
Constant	Random	Deterministic	**P**-κ_C-c_R-k_D
Linear	Random	Deterministic	**P**-κ_L-c_R-k_D
Constant	Random	Random	**P**-κ_C-c_R-k_R
Linear	Random	Random	**P**-κ_L-c_R-k_R
Random Field,b=2	Random Field,b=2	Random Field,b=2	**P**-κ_{RF}-c_{RF}-k_{RF2}
Random Field,b=4	Random Field,b=4	Random Field,b=4	**P**-κ_{RF}-c_{RF}-k_{RF4}
Random Field,b=8	Random Field,b=8	Random Field,b=8	**P**-κ_{RF}-c_{RF}-k_{RF8}

Porous analyses performed.

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

- κ_L ➔ κ(z=0)=0,008686 and $R = $ κ(z=h)/κ(z=0) is random with $R_{\text{mean}} = 0{,}469$ and $CV_R = 0{,}25$

- κ_C ➔ κ_μ =0,004074 and $CV = 0{,}25$

- c_R ➔ Mean value of friction angle 23° and standard deviation of 2° and $c = \frac{\sqrt{\frac{2}{3}}(6\sin(\varphi))}{3-sin(\varphi)}$

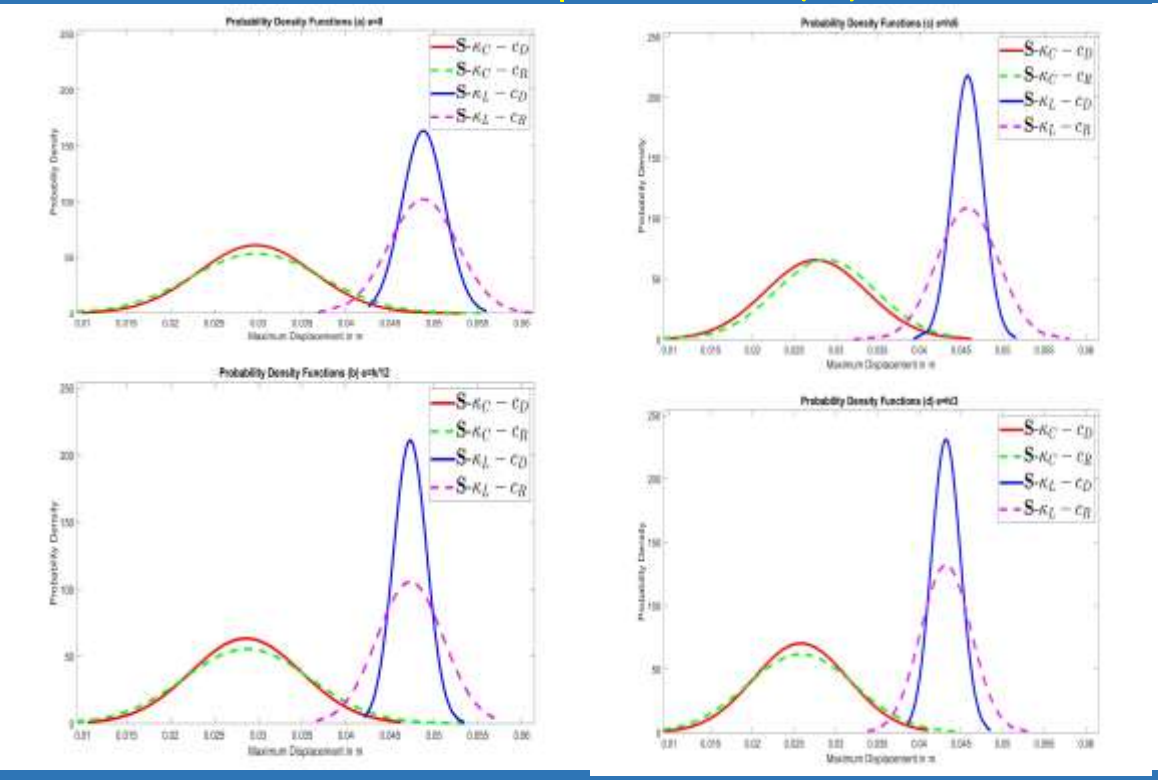
- c_C ➔ c=0,7336 for friction angle of 23°

- κ_{RF} ➔ Mean value=0,008686, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100
-
- c_{RF} ➔ Mean value= 0,7336, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100
-
- k_{RF} ➔ Mean value=$10^{-8}\,\frac{m^3 s}{Mgr}$, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100

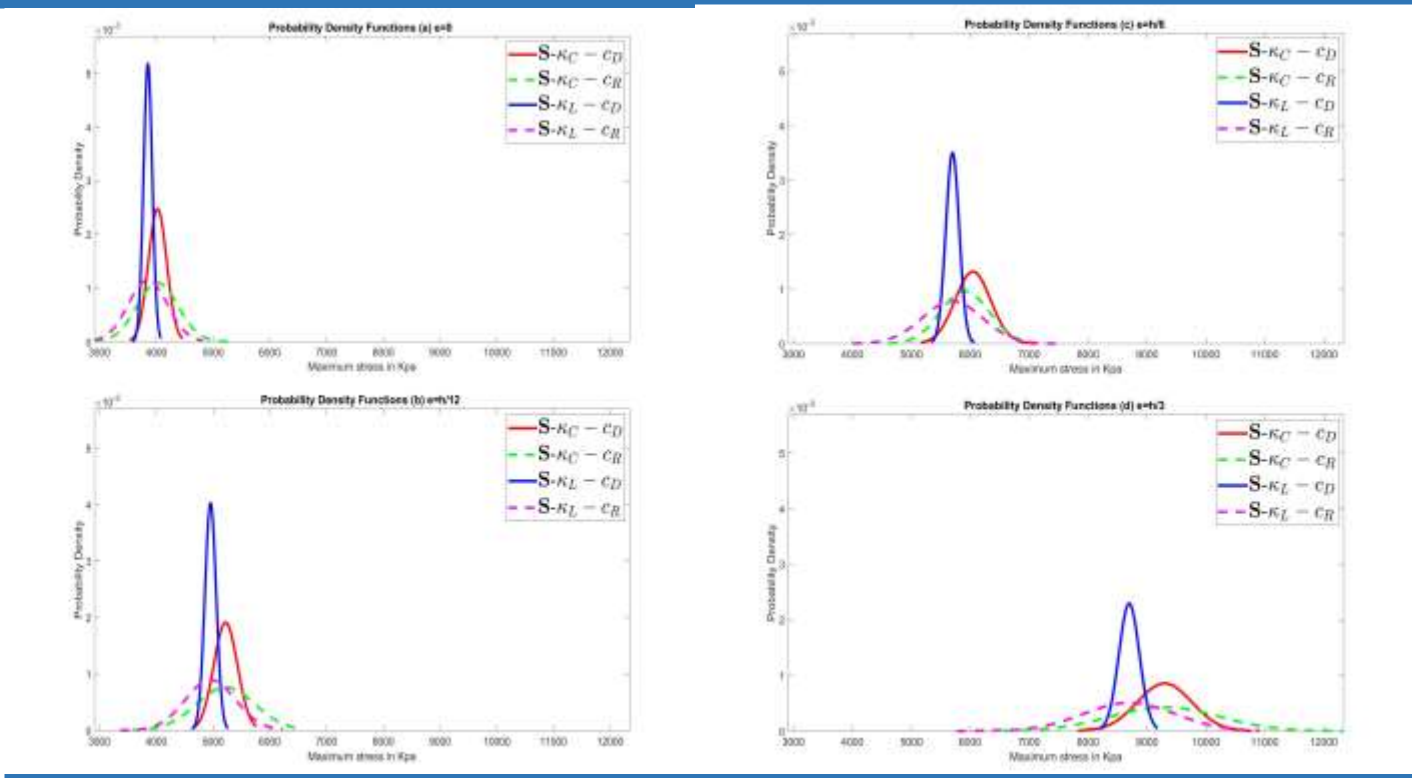
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Solid analyses

Failure displacements (m)



Failure load (Kpa)



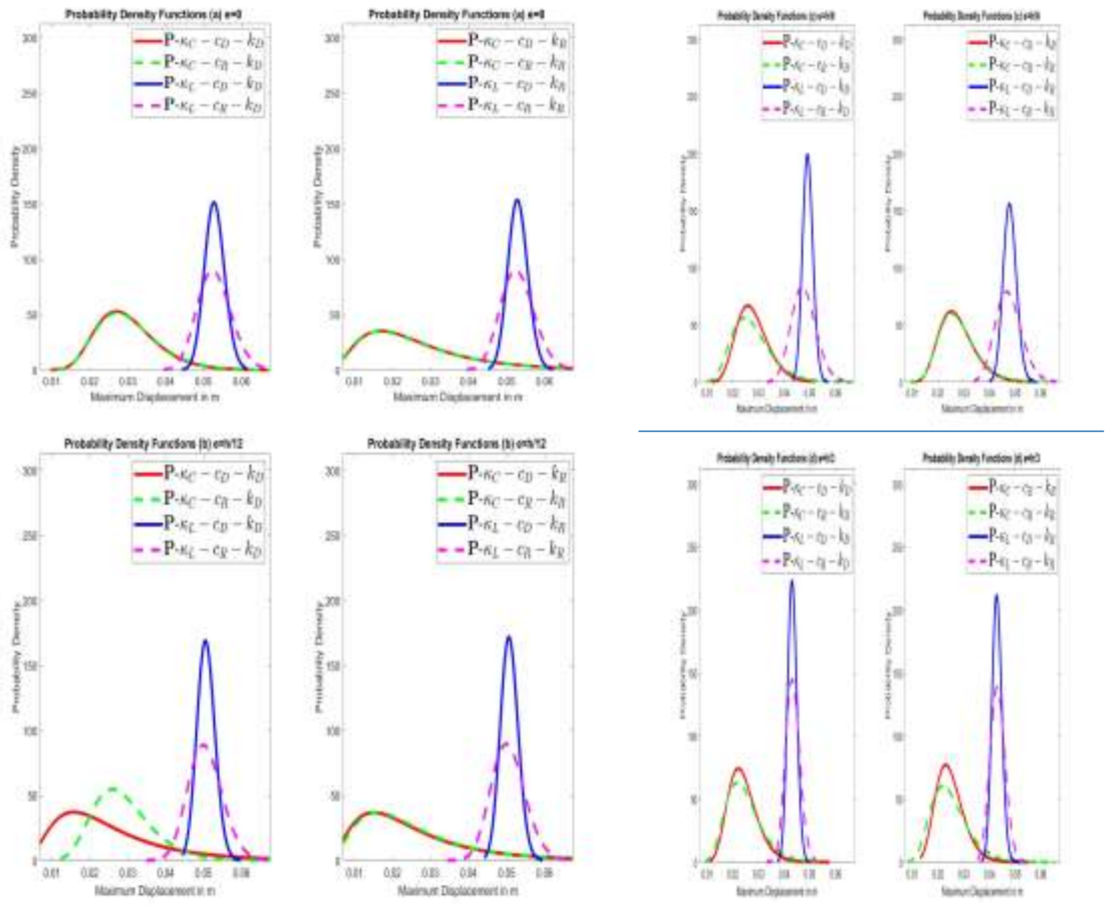
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

- When κ_L distribution is assumed, larger mean failure displacement, larger mean failure loads, smaller failure displacement and larger failure loads CV is obtained in relation with κ_c . The largest uncertainty for failure load is about 40% the uncertainty of the input while for the failure displacements is about the same of the input variability
- Critical spatial distribution for mean value and CV for both output variables is κ_C
- As the eccentricity increases the mean failure load increases and the mean settlement rotation decreases
- Justification➔ In κ_L the upper layers of the soil are more compressible though with less variability so the strains are expected with less variability and so it is expected the output displacement. When the constant distribution for the compressibility is assumed more integration points have the same or similar stiffness thus leading to larger failure loads.

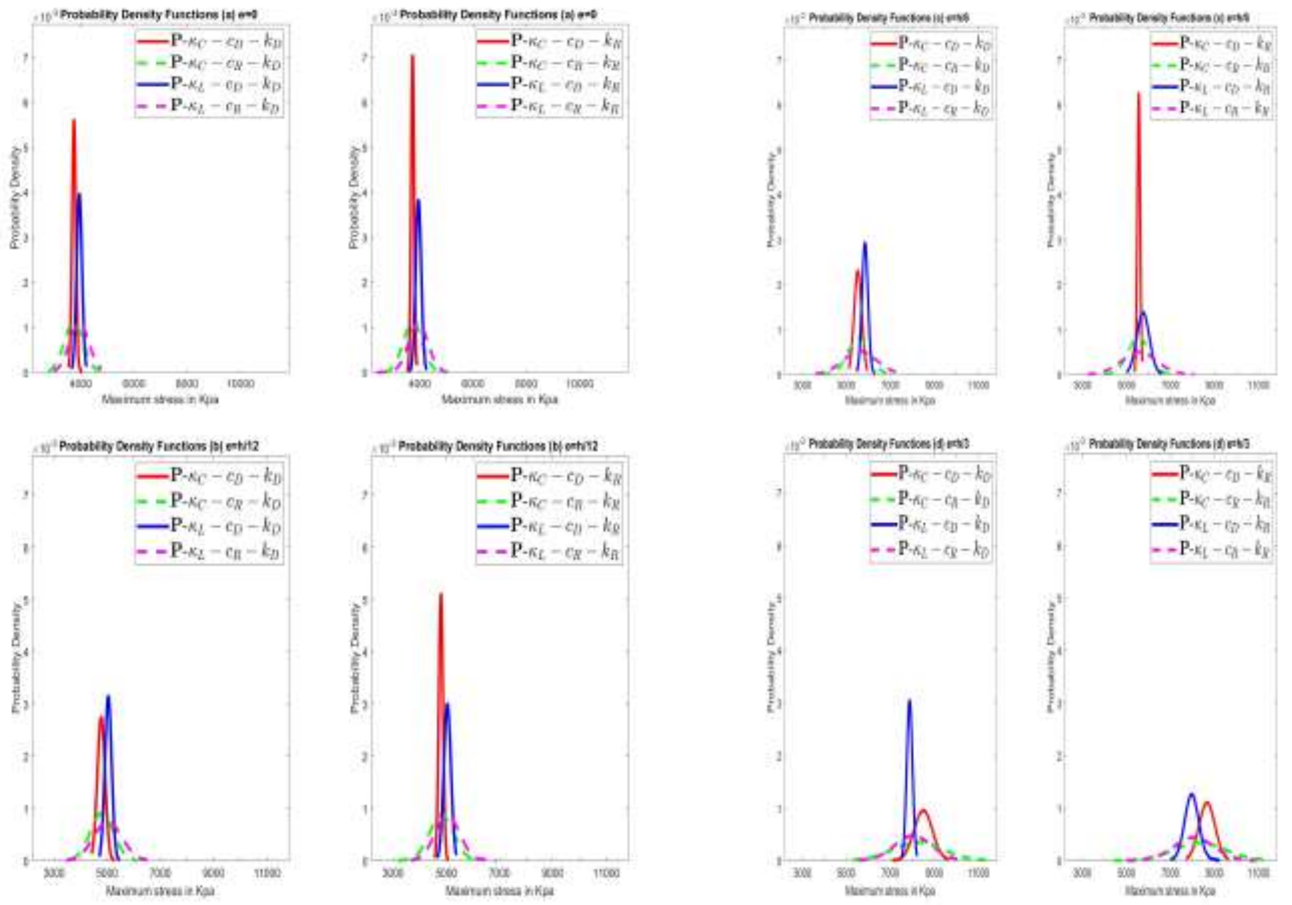
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Porous analyses with deterministic shape functions for κ, c, k

Failure displacements (m)



Failure load (Kpa)



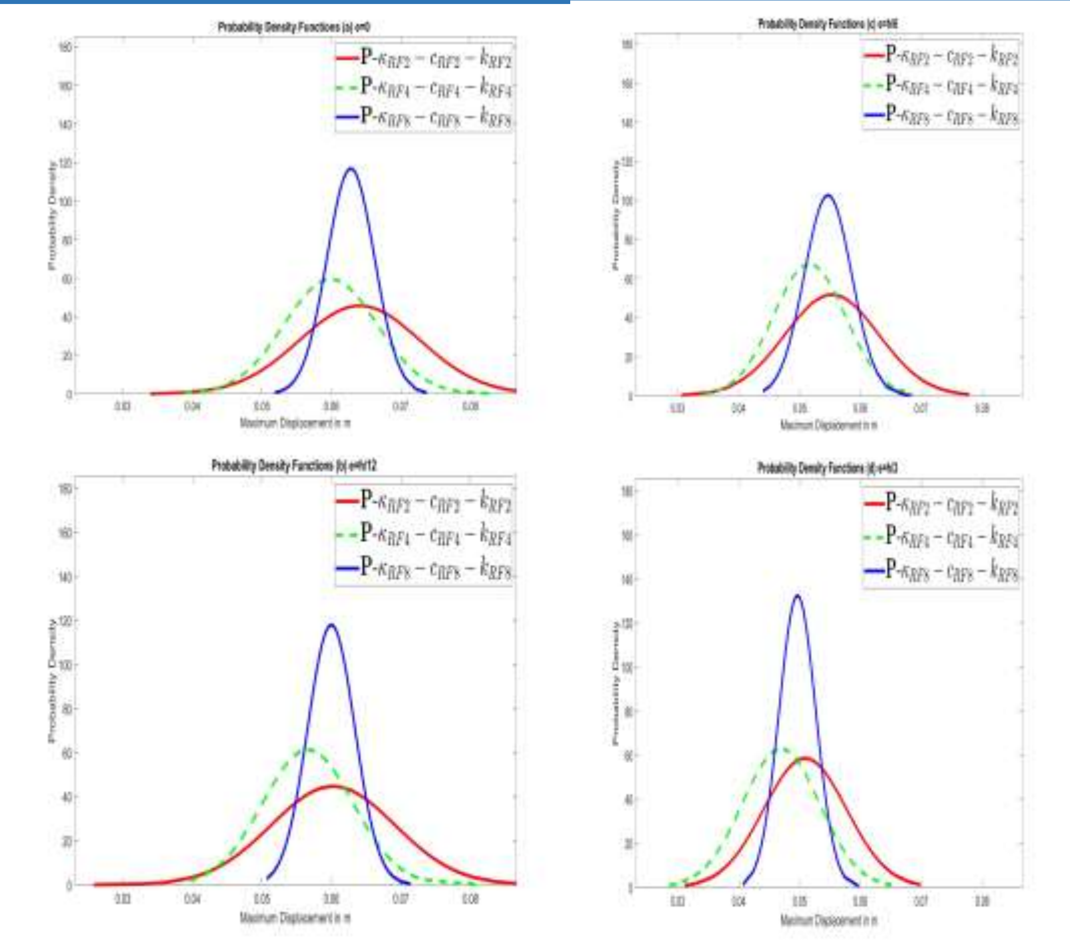
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

- The CV of failure loads is less dependent by the eccentricity, unlike the displacements and rotations.
- Maximum CV of failure load in porous analyses (κc case) is 44 % smaller than the input CV and for failure displacements 2,6 times greater than the input CV
- Porous analyses➔ Important variability reduction for failure maximum stress whilst for failure displacements in the case of the constant distribution for the compressibility factor significant variability increase occurs and the rotation of the footing has output uncertainty in the vicinity of the input variability
- Justification ➔ Bulk modulus K_b is a function of mean stress (Poroelasticity). So K_b , in porous analyses is expected with smaller values and smaller uncertainrty. Similar conclusions can be made for the failure displacements since there is no tensile strength of the soil point and consequently there is smaller surface of the BSE leading to the aforementioned results

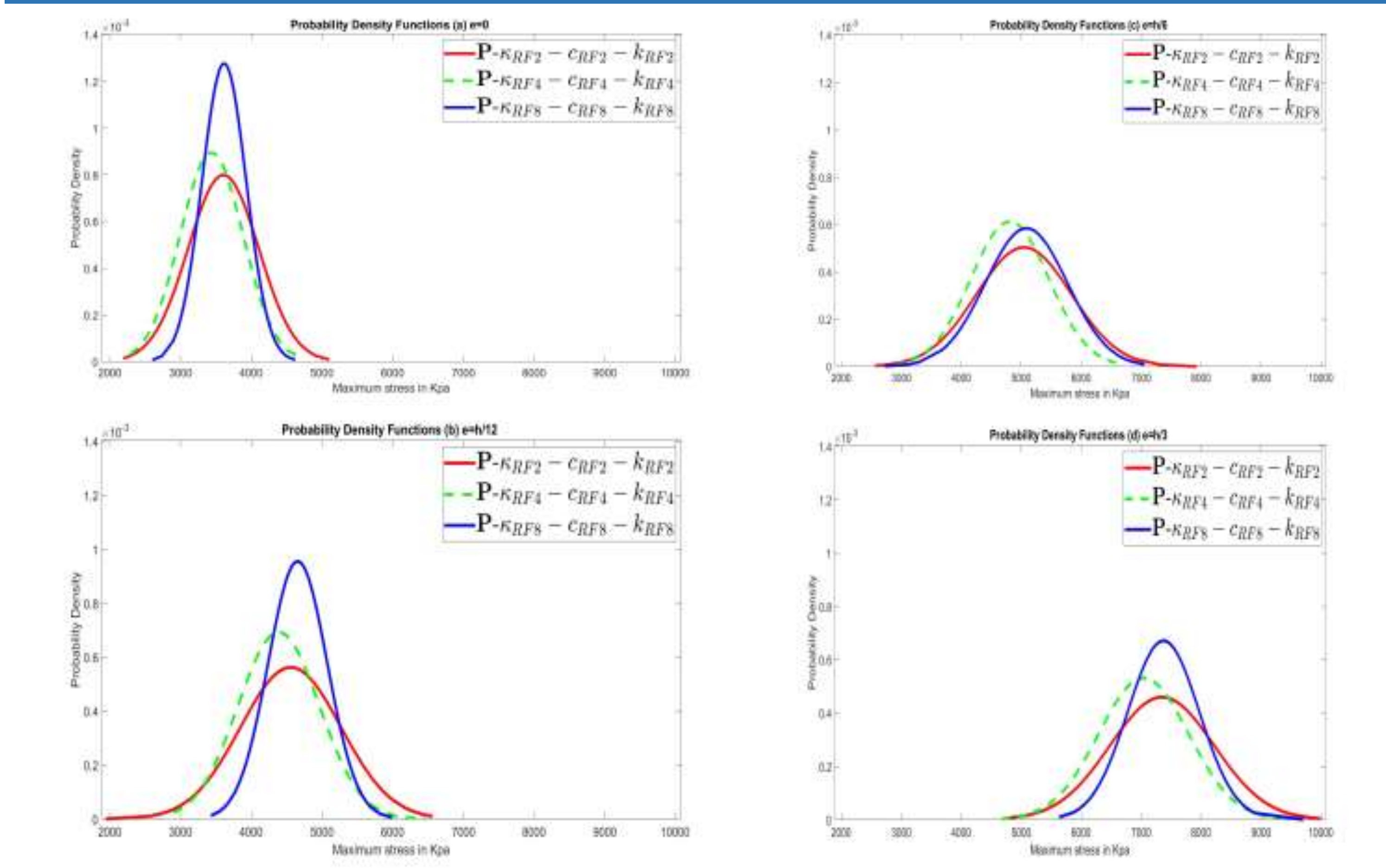
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Porous analyses with random field representation for all stochastic material variables

Failure displacements (m)



Failure load (Kpa)



Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

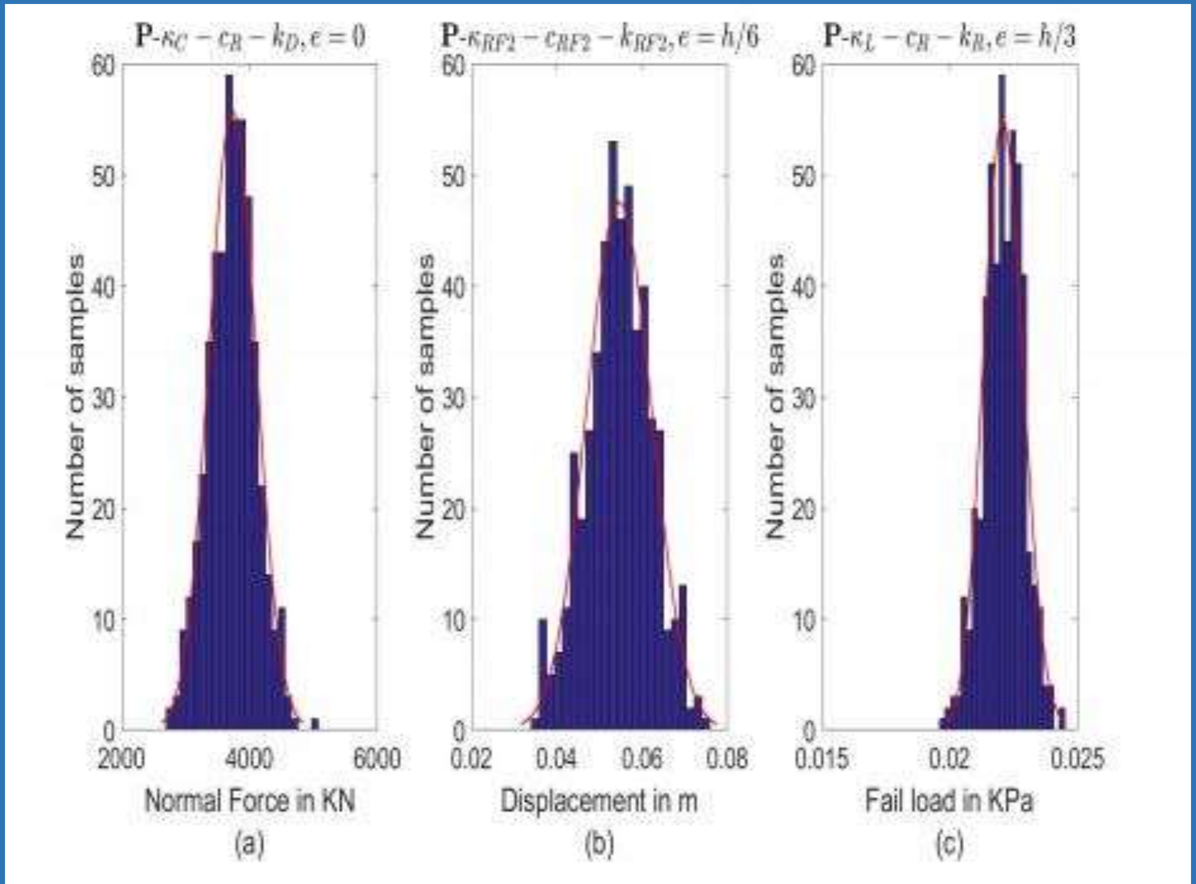
- Maximum CV of the output for failure load and displacements is about 36% lower than the uncertainty of the input while for rotation of the footing is equal to the variability of the input.
- Mean values of failure load and failure displacements slightly deviate from the corresponding mean values in the analyses with deterministic values for the material variables. This deviation is up to 17% for the forces and in the vicinity of 20% for the displacements.
- In the case of porous random field analyses the increase of the correlation length for reduces the CV of the output for stresses and displacements whilst for the rotation of the settlement at failure the critical correlation length is 4 m.
- Justification ➔ The integration point failure may be «from the wet side» (from the left side of the vertical halfaxis of the ellipse) or «from the dry side» (from the right side of the vertical halfaxis of the ellipse) consequently a large change of the value of c may incorporate very large deviation of the stress point of failure leading to the aforementioned results.

Numerical tests on stochastic consolidation with random linear and non linear material-Analysis of the results (Kolmogorov Smirnov Test)

- Assumption → The output displacement follows the truncated normal distribution.
- Justification from Histograms → Graphically this holds
- Justification from numerical test → Kolmogorov Smirnov test for a sample following a distribution.
- The largest absolute difference of the theoretical and the numerical CDF is compared to the critical value. Since it is less than the critical value the null hypothesis holds and the sample follows the truncated normal distribution. Therefore the null hypothesis at the 5% significance level is satisfied to the randomly selected analyses presented to the histograms
- Despite the material non linearity the output displacement still has the Gaussian nature of the randomness

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Kolmogorov-Smirnov Test



Largest Absolute Difference	Figure (a)	Figure (b)	Figure (c)	Critical
Significance level 5%	0,0821	0,0972	0,1121	0,13851

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Solid Analyses

- The distribution for the critical state line inclination cR provides the larger uncertainty and the smallest minimum values of failure in both hydrostatic and deviatoric components.
- The same distribution maximizes the uncertainty of the strains which are in the order of magnitude of 12 ‰ for the linear distribution for κ and 5 ‰ for the constant distribution for κ
- In most cases the deviatoric failure occurs.
- For eccentricity not equal to zero the critical Gauss point is (3,21, 2,21, 3,79)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Porous analyses with deterministic shape functions

- cR case provides the maximum uncertainty and smallest minimum values of failure stresses when the eccentricity is not zero.
- The eccentricity zero gives the largest uncertainty on the failure strains in both compoments
- Smaller eccentricities and large eccentricities with linear distribution of the compressibility factor provide volumetric failure while large eccentricities and constant distribution for κ provide volumetric failure
- Critical Gauss point is (3,21, 2,79, 3,79) in all eccentricities except h/6 (κL cR case)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Porous random field analyses

- The correlation length 2 m provides the maximum uncertainty at stresses.
- The correlation length 4 m provides the largest variability at stresses.
- When the eccentricity is not zero mainly the distortional failure occurs.
- When the values for the eccentricity is small the critical integration point is (3,21, 2,79, 3,79) and as the eccentricity increases the critical Gauss point becomes (3,21, 2,21, 3,79)

Conclusions

- Failure load, failure displacements and failure spline and corresponding stresses-strains follow the Gaussian distribution despite the excessive material non linearity
- The compressibility factor κ plays an significant role especially when it has a constant distribution. Similar conclusions apply for the plasticity variable c. Permeability has a smaller influence to the uncertainty of the output.
- The amplification of the uncertainty in displacements can be up to 2,6 times larger
- The random field processes provide larger mean failure strains
- In general, for small eccentricities the distortional failure occurs and as the eccentricity increases if the compressibility factor varies with a constant distribution over depth the volumetric failure occurs.
- In the majority of the cases the integration point (3,21 , 2,79 , 3,79) may be considered the onset of the Meyerhoff spline.

Thank you all for your attention.
May you stay safe from the pandemic and soon enough to be able to conference live .
Questions?