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Uncertainty quantification of failure of clays with a modified Cam Clay yield criterion and Stochastic FEM

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Presentation contents

- Introduction (Scope-goal)
- Presentation of the theory (Porous media, material constitutive model, stochastic finite element analysis)
- Numerical tests on stochastic consolidation with random linear and non linear material properties
- Conclusions

Introduction

- Porous consolidation of clays. ➔ Equillibrium equations of solid coupled with equations of fluid flow

- For lower frequency excitations ➔ u-p formulation which requires less computational effort

- The soil behavior is described with the compressibility factor κ, the critical state line inclination of the soil c and the permeability k. In this context, Modified Cam Clay models are implemented.

Introduction

- In the stochastic finite element method there are two ways of providing the spatial distribution of the input stochastic variables

1. The nodal points are considered as random variables and deterministic shape functions are used for providing the material spatial distribution
2. The spectral representation or the Karhunen Loeve sum, can be applied for providing a random field of the variable under consideration.

Introduction

- In this work the material yield model proposed by Kavvadas and Amorosi is adopted → accurate and reliable material model

- Valid quantitative results for predicting the response for porous consolidation of clays under uncertainty

- The goal is to quantify the uncertainty of the output displacement in relation to the variability of the input, the spatial distribution of the material variables and the soil depth

Dynamic soil-pore-fluid interaction-The Biot problem

- Equation of equilibrium for the soil-fluid mixture

$$\boldsymbol{S}^T\boldsymbol{\sigma} - \rho\ddot{\mathbf{u}} - \rho_f(\dot{\boldsymbol{w}} + \boldsymbol{w}\nabla^T\boldsymbol{w}) + \rho\boldsymbol{b} = \mathbf{0}$$

- Equation of balance of the fluid

$$-\nabla p - \boldsymbol{R} - \rho_f\ddot{\mathbf{u}} - \frac{\rho_f(\dot{\boldsymbol{w}} + \boldsymbol{w}\nabla^T\boldsymbol{w})}{n} + \rho_f\boldsymbol{b} = \mathbf{0}$$

- Flow conservation equation

$$\nabla^T\boldsymbol{w} + \left(1 - \frac{K_T}{K_s}\right)\boldsymbol{I}\dot{\boldsymbol{\varepsilon}} + \dot{p}\left(\frac{n}{K_f} + \frac{1-n}{K_s}\right) + \frac{n\dot{\rho}_f}{\rho_f} + \dot{\boldsymbol{s}} = \mathbf{0}$$

- Darcy law ➔ $\boldsymbol{kR} = \boldsymbol{w}$

- Stress-Strain material law and pore pressure stress equation

$$\boldsymbol{d\sigma}' = \boldsymbol{Dd\varepsilon}_{\boldsymbol{el}}\,, \boldsymbol{\sigma}' = \boldsymbol{\sigma} + p$$

- Boundary conditions of known pressures, displacements, water velocity and stresses in specific parts of the boundary.

Dynamic soil-pore-fluid interaction-The u-p problem

- Equation of equilibrium for the soil-fluid mixture

$$\boldsymbol{S}^T\boldsymbol{\sigma} - \rho\ddot{\mathbf{u}} + \rho\boldsymbol{b} = \mathbf{0}$$

- Flow conservation equation

$$\nabla^T\boldsymbol{k}(-\nabla p - \rho_f\ddot{\mathbf{u}} + \rho_f\boldsymbol{b}) + \left(1 - \frac{K_T}{K_s}\right)\boldsymbol{I}\dot{\boldsymbol{\varepsilon}} + \dot{p}\left(\frac{n}{K_f} + \frac{1-n}{K_s}\right) + \dot{\mathbf{s}} = \mathbf{0}$$

- Stress-Strain material law and pore pressure stress equation

$$\boldsymbol{d\sigma}' = \boldsymbol{D}\boldsymbol{d}\boldsymbol{\varepsilon}_{\boldsymbol{el}}\,, \boldsymbol{\sigma}' = \boldsymbol{\sigma} + p$$

- Boundary conditions of known pressures, displacements, water velocity and stresses in specific parts of the boundary.

- This formulation is valid in low frequency excitations in relation with the eigenfrequency of the soil and in static problems.

Dynamic soil-pore-fluid interaction-FEM simulation

- Equation of discretized u-p formulation

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{Ku} = \boldsymbol{F}$$

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C_1} & \boldsymbol{0} \\ \boldsymbol{Q_c^T} & \boldsymbol{S} \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} \boldsymbol{K_1} & \boldsymbol{-Q_c} \\ \boldsymbol{0} & \boldsymbol{H} \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} \boldsymbol{F_1} \\ \boldsymbol{0} \end{bmatrix}, \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix}$$

Where $\boldsymbol{M_1}$, $\boldsymbol{C_1}$, $\boldsymbol{K_1}$ are the standard mass, damping and stiffness matrices respectively and $\boldsymbol{Q_c}$, $\boldsymbol{H}$, $\boldsymbol{S}$ are the coupling, permeability and saturation matrices respectively and are expressed as

$$\boldsymbol{Q_c} = \int_V \boldsymbol{B^T I N_p} \, dV, \boldsymbol{H} = \int_V (\boldsymbol{\nabla N_p})^{\boldsymbol{T}} \boldsymbol{k} (\boldsymbol{\nabla N_p}) \, dV, \boldsymbol{S} = \int_V \boldsymbol{N_p^T N_P} \left(\frac{n}{K_f} + \frac{1-n}{K_s} \right) dV$$

The material yield stress model

- Bond Strength Envelope (BSE)

$$\frac{1}{c^2}\boldsymbol{s}:\boldsymbol{s} + (p_h - a)^2 - a^2 = 0$$

- Plastic Yield Envelope (PYE)

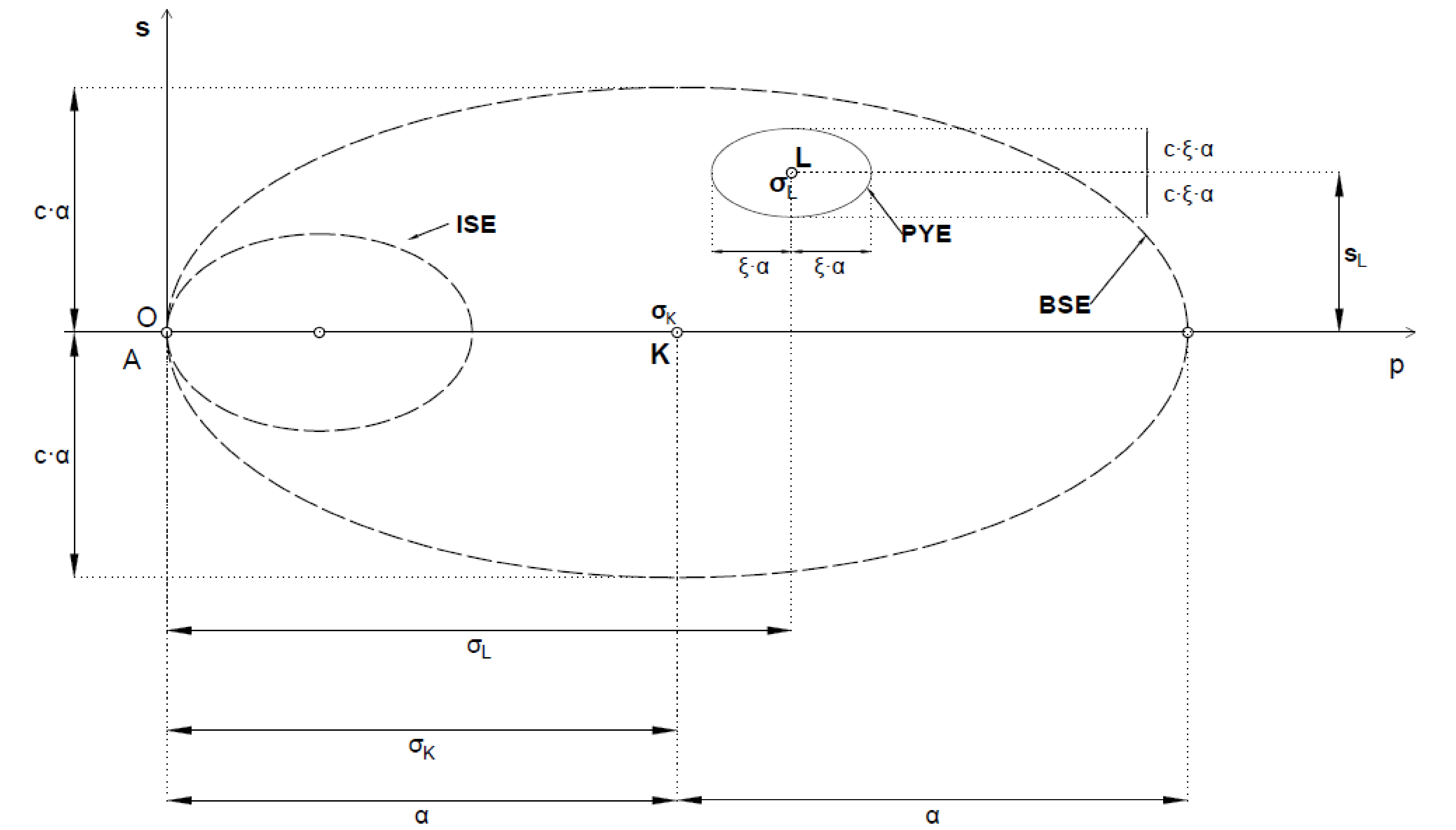
$$\frac{1}{c^2}(\boldsymbol{s} - \boldsymbol{s}_L):(\boldsymbol{s} - \boldsymbol{s}_L) + (p_h - p_L)^2 - (\xi a)^2 = 0$$

- Intrinsic Strength Envelope (ISE)

$$\frac{1}{c^2}\boldsymbol{s}:\boldsymbol{s} + (p_h - a)^2 - a^2 = 0$$

- The PYE is always inside BSE and ISE is the smallest possible BSE

The material yield stress model



Graphical representation of the constitutive model

The Karhunen Loeve Series

- For a random field with zero mean and an autocovariance function

$$C_h = \sigma_d^2 e^{\frac{\Delta X}{b}}$$

The solution of the integrodifferential Fredholm problem provide analytical expressions for the eigenvalues λ and sinusoidal expressions for eigenfunctions Φ. Therefore a robust equation of the random field and is expressed as follows

$$H(\boldsymbol{x}, \omega) = \mu(\boldsymbol{x}) + \sum_{1}^{M} \sqrt{\lambda_i}\, \Phi_{\mathrm{i}}(\mathbf{x}) \xi_{\mathrm{i}}(\omega)$$

For M number of eigenfunctions chosen to achieve minimum error and $\xi_{\mathrm{i}}(\omega)$ a set of standard normal random variables.

The truncated normal distribution

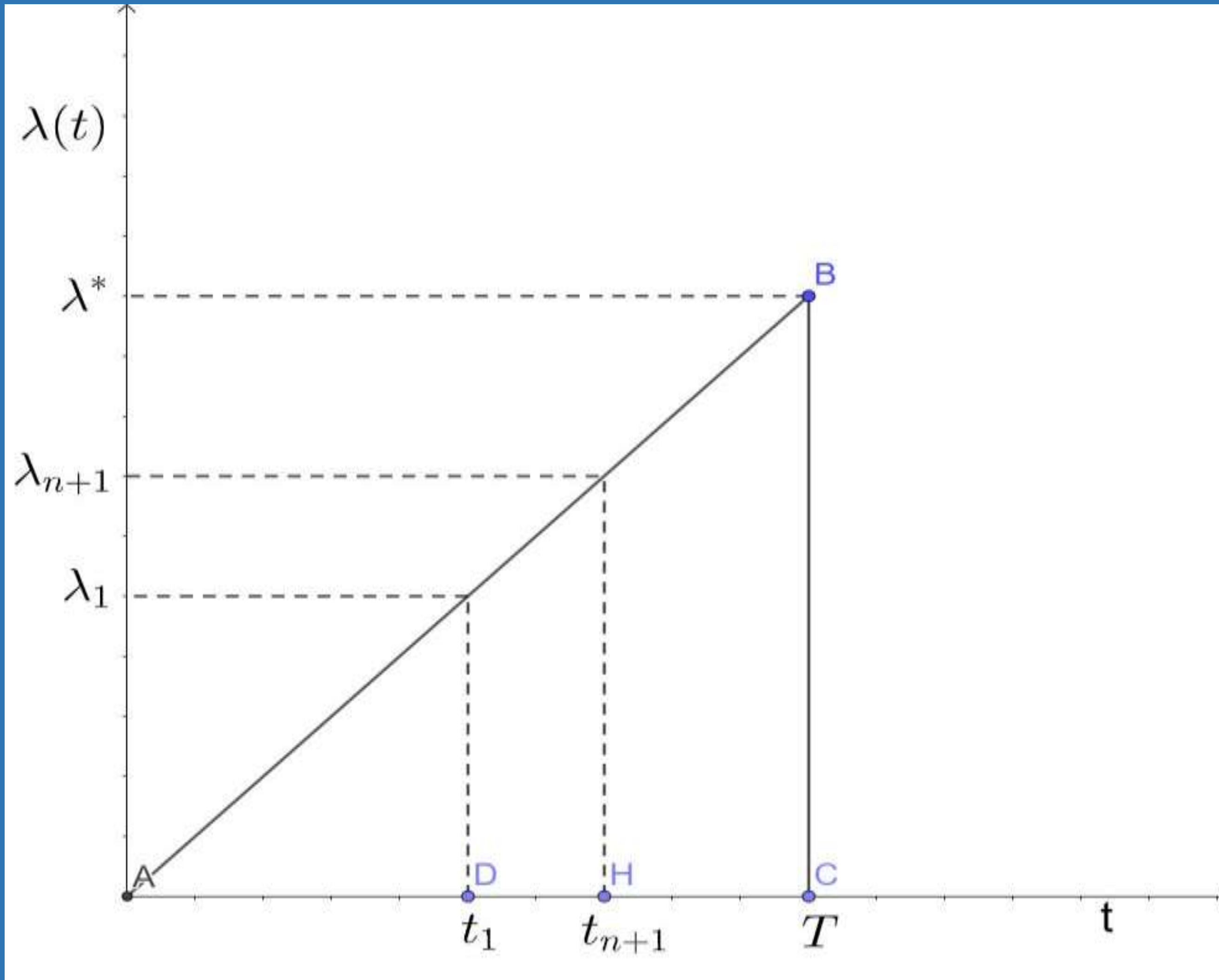
- If the domain of definition of the probability is the [a, b] and we know the normal distribution moments of the PDF in $[-\infty, \infty]$ (μ, σ_d) then the PDF in [a, b] is expressed as

$$g_1(x) = \frac{\varphi(\mathrm{X}_0)}{\sigma_d\left(\Phi(\mathrm{B})-\Phi(\mathrm{A})\right)}$$

Where ϕ and Φ represent the standard normal probability and cumulative distribution function respectively. The X_0 , B, A are the normalized coordinates for x, a, b respectively. The mean value and standard deviation of $g_1(x)$ are functions of the values (μ, σ_d) and the values of ϕ and Φ in A and B

Algorithm for the determination of the failure load in the case of a ramp loading time function

- Let the ramp load function be defined. For determining the factor λ* at the time T a recursive relation is proposed:
- $\lambda_{n+1} = 0{,}5(\frac{\lambda_n t_n}{T} + \lambda_n)$
- The difference between two adjacent iterations at the infinite tends to 0 consequently convergence is achieved.

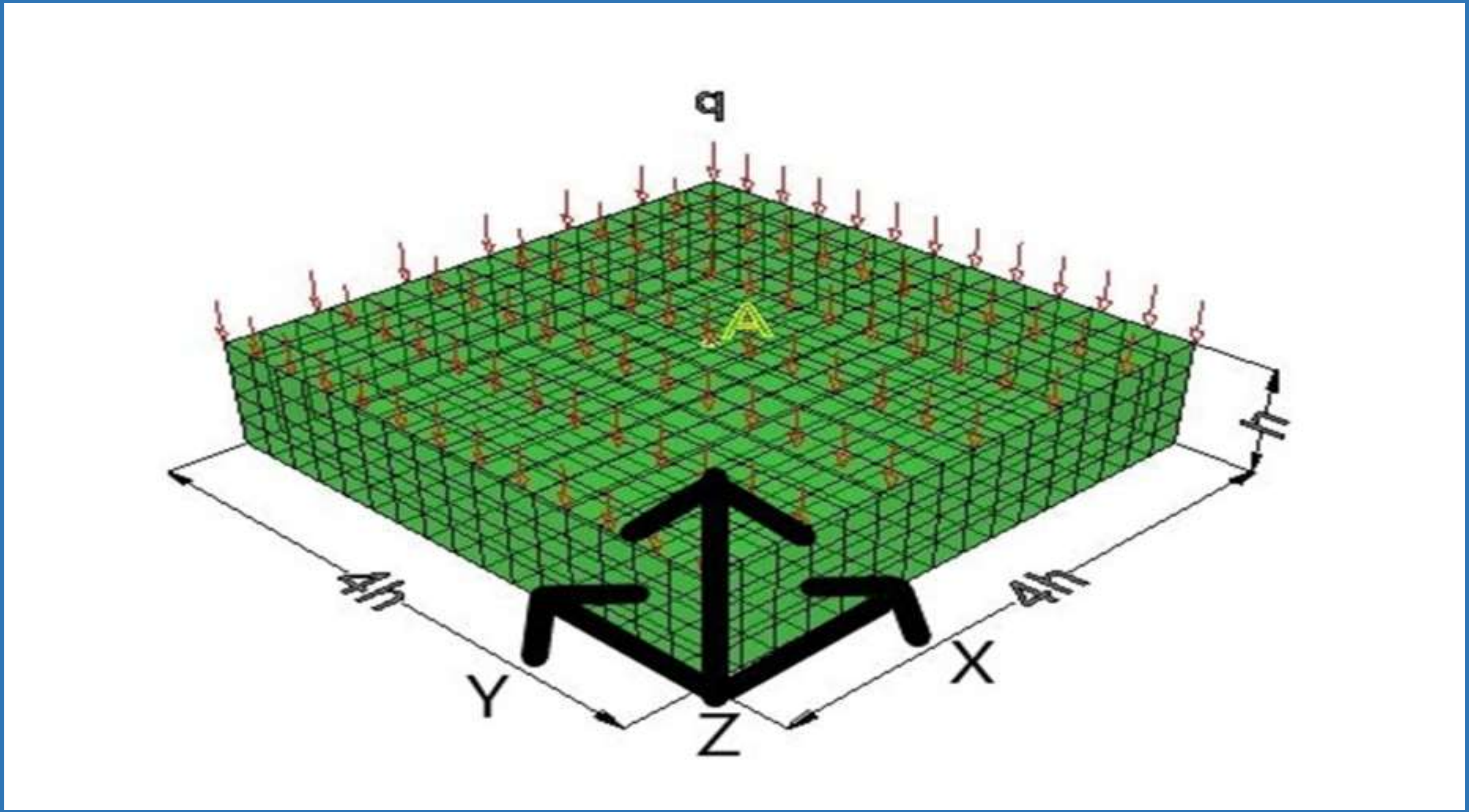


Algorithm for the determination of the failure load in the case of a ramp loading time function

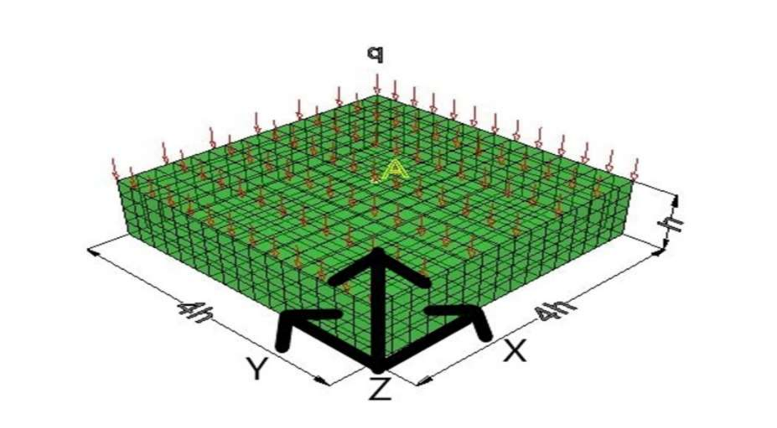
- Only one initial trial guess
- Smaller number of iterations for convergence
- Computational time reduction in comparison with the bisection method in the vicinity of 35%
- Relative error less than 1%

	Bisection Algorithm	Bisection Algorithm	Bisection Algorithm	Proposed Algorithm	Absolute percentage difference
Initial value of failure stress (KPa)	400	400	400	400	
Initial value of safety stress (KPa)	20	50	100	-	
Convergence Tolerance	0,01	0,01	0,01	0,01	
Number of trials for convergence	6	6	6	5	
Displacement of failure at convergence (m)	0,18283	0,18294	0,18300	0,18288	0,066
Load of failure at convergence (KPa)	362,797	363,085	363,671	365,687	0,551
Computational time (mins)	10388	9138	9406	6938	35,57

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description



Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description



- Porous problems and solid problems
- 3 depths h=20,40,50 m and dimensions X=Y=4h and Uniform vertical load 150 Kpa
- 8 node Hexa with linear shape functions for u and p
- Initial stresses ➔ Geostatic vertical stresses and horizontal stresses=0,85* vertical stresses
- Boundary conditions ➔ **u**(z=h)=**0**
- Deterministic calibration parameters, Poisson ratio, plasticity hardening parameters
- 100 samples of Monte Carlo simulations considering the Latin Hypercube Sampling

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

κ	c	Abbreviation
Constant	Constant	**S**-κ_C-c_C
Linear	Constant	**S**-κ_L-c_C
Constant	Random	**S**-κ_C-c_R
Linear	Random	**S**-κ_L-c_R

Solid analyses performed.

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

κ	c	k	Abbreviation
Constant	Constant	Random Field,b=75	**P**-$κ_C$-c_C-k_{RF75}
Linear	Constant	Random Field,b=75	**P**-$κ_L$-c_C-k_{RF75}
Constant	Constant	Random Field,b=100	**P**-$κ_C$-c_C-k_{RF100}
Linear	Constant	Random Field,b=100	**P**-$κ_L$-c_C-k_{RF100}
Constant	Random	Random Field,b=75	**P**-$κ_C$-c_R-k_{RF75}
Linear	Random	Random Field,b=75	**P**-$κ_L$-c_R-k_{RF75}
Constant	Random	Random Field,b=100	**P**-$κ_C$-c_R-k_{RF100}
Linear	Random	Random Field,b=100	**P**-$κ_L$-c_R-k_{RF100}
Random Field,b=75	Random Field,b=75	Random Field,b=75	**P**-$κ_{RF}$-c_{RF}-k_{RF75}
Random Field,b=100	Random Field,b=100	Random Field,b=100	**P**-$κ_{RF}$-c_{RF}-k_{RF100}

Porous analyses performed.

Numerical tests on stochastic consolidation with random linear and non linear material properties-Problem description

- κ_L ➔ κ(z=0)=0,008686 and $R = $ κ(z=h)/κ(z=0) is random with $R_{\text{mean}} = 0{,}469$ and $CV_R = 0{,}25$
- κ_C ➔ κ_μ =0,004074 and $CV = 0{,}25$
- c_R ➔ Mean value of friction angle 23° and standard deviation of 2° and $c = \frac{\sqrt{\frac{2}{3}}(6\sin(\varphi))}{3-sin(\varphi)}$
- c_C ➔ c=0,7336 for friction angle of 23°
- κ_{RF} ➔ Mean value=0,008686, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100
-
- c_{RF} ➔ Mean value= 0,7336, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100
-
- k_{RF} ➔ Mean value=$10^{-8}\,\frac{m^3 s}{Mgr}$, exponential autocorrelation function $CV = 0{,}25$, b=75 and 100

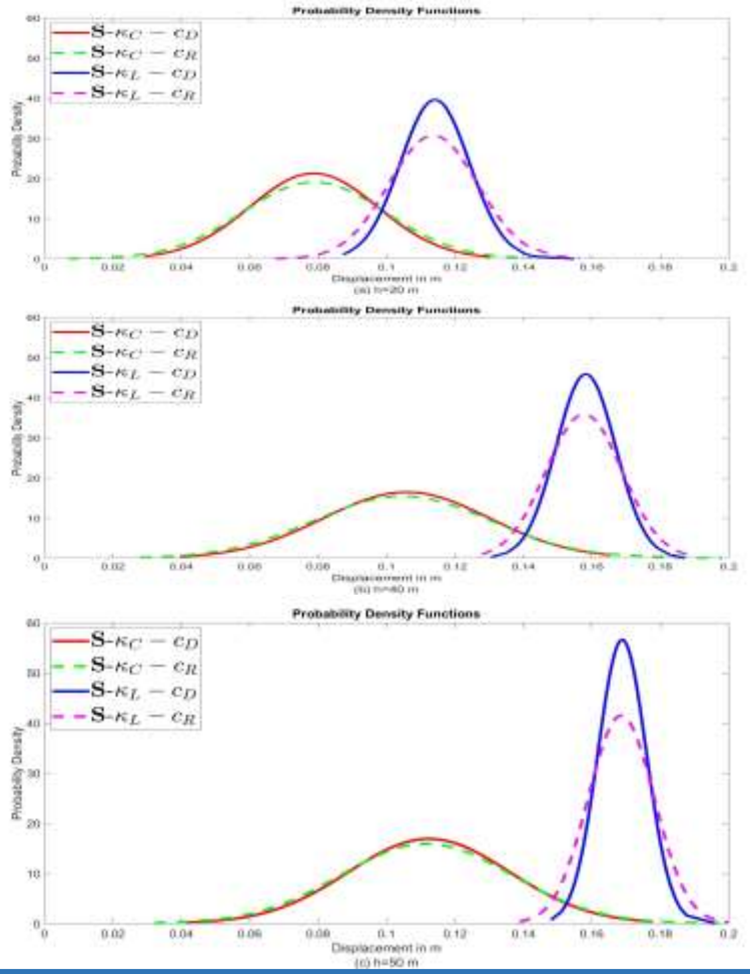
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Solid analyses

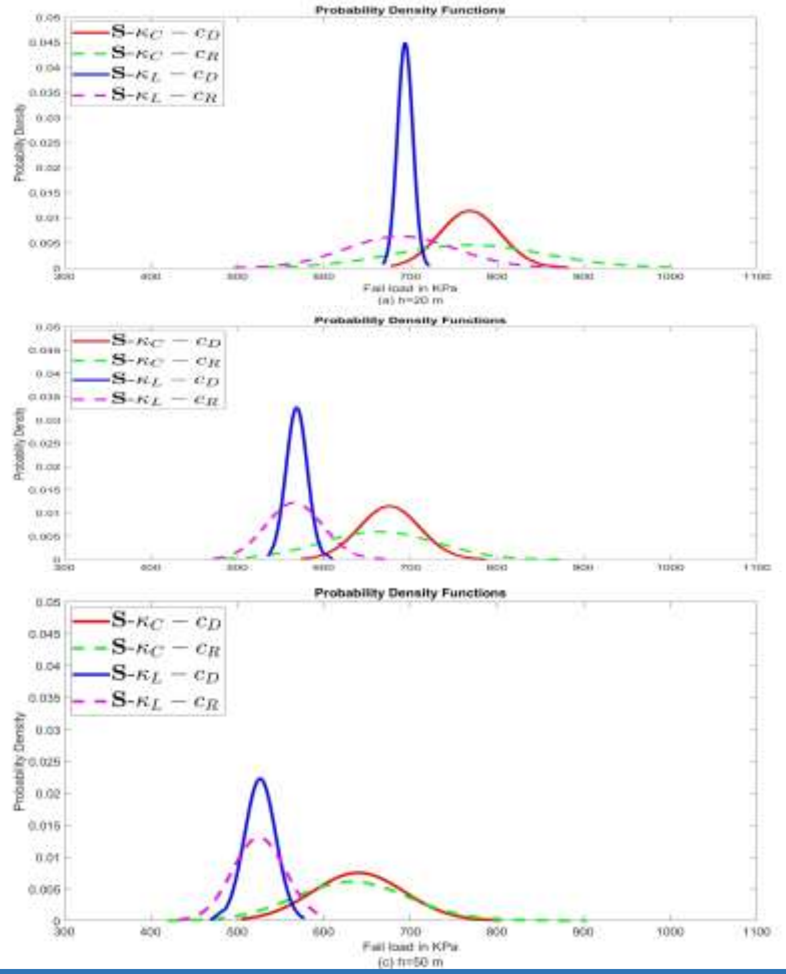
	h=20 m, q_{fail}				h=20 m, u_{fail}			
S−	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	768,63	768,25	693,75	688,44	0,0787	0,0788	0,1141	0,1136
Stdev	33,95	84,65	8,61	61,13	0,0181	0,0202	0,0098	0,0126
CoV	0,0442	0,1102	0,0124	0,0888	0,2305	0,2565	0,0855	0,1106
MAX	854	948	712	795	0,1122	0,1185	0,1321	0,1386
MIN	715	619	679	565	0,0392	0,0403	0,0928	0,0945
$\frac{MAX}{MIN}$	1,19	1,53	1,05	1,41	1,19	1,53	1,05	1,41

	h=40 m, q_{fail}				h=40 m, u_{fail}			
S−	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	675,75	663,25	568,50	565,56	0,1058	0,1046	0,1582	0,1577
Stdev	33,70	64,68	11,82	31,66	0,0235	0,025	0,0084	0,0108
CoV	0,0499	0,0975	0,0208	0,0560	0,2218	0,2390	0,0533	0,0683
MAX	744	792	605	624	0,1470	0,1511	0,1743	0,1789
MIN	608	546	552	497	0,0531	0,0548	0,1417	0,1436
$\frac{MAX}{MIN}$	1,22	1,45	1,10	1,26	2,77	2,76	1,23	1,25

	h=50 m, q_{fail}				h=50 m, u_{fail}			
S−	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	639,81	632,13	526,31	524,94	0,1124	0,1116	0,1689	0,1686
Stdev	51,34	63,2	17,34	29,6	0,0229	0,0243	0,0068	0,0093
CoV	0,0802	0,1000	0,0329	0,0564	0,2034	0,2174	0,0404	0,0553
MAX	738	778	580	600	0,1527	0,1565	0,1826	0,1871
MIN	554	543	504	476	0,0586	0,0606	0,1571	0,1563
$\frac{MAX}{MIN}$	1,33	1,43	1,15	1,26	2,61	2,58	1,16	1,20



Failure displacements (m)



Failure load (Kpa)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

- When κ_L distribution is assumed, larger mean failure displacement and smaller CV is obtained in relation with κ_c . The largest uncertainty for failure load is about half the uncertainty of the input while for the failure displacements is about the same of the input variability
- Critical spatial distribution for mean value and CV for both output variables is κ_C
- Justification➔ In κ_L the upper layers of the soil are more compressible though with less variability so the strains are expected with less variability and so it is expected the output displacement. When the constant distribution for the compressibility is assumed more integration points have the same or similar stiffness thus leading to larger failure loads.

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Porous analyses with deterministic shape functions for κ and c

h=20 m	$k_{RF}75$-q_{fail}				$k_{RF}100$-q_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	516,21	514,68	513,80	511,97	422,48	514,89	370,39	512,99
Stdev	28,22	68,09	36,40	62,66	30,23	68,05	32,77	63,43
CoV	0,0547	0,1323	0,0708	0,1224	0,0715	0,1322	0,0885	0,1237
MAX	595	630	572	626	505	629	435	631
MIN	480	385	429	422	371	385	326	423
$\frac{MAX}{MIN}$	1,24	1,64	1,33	1,48	1,36	1,63	1,34	1,49

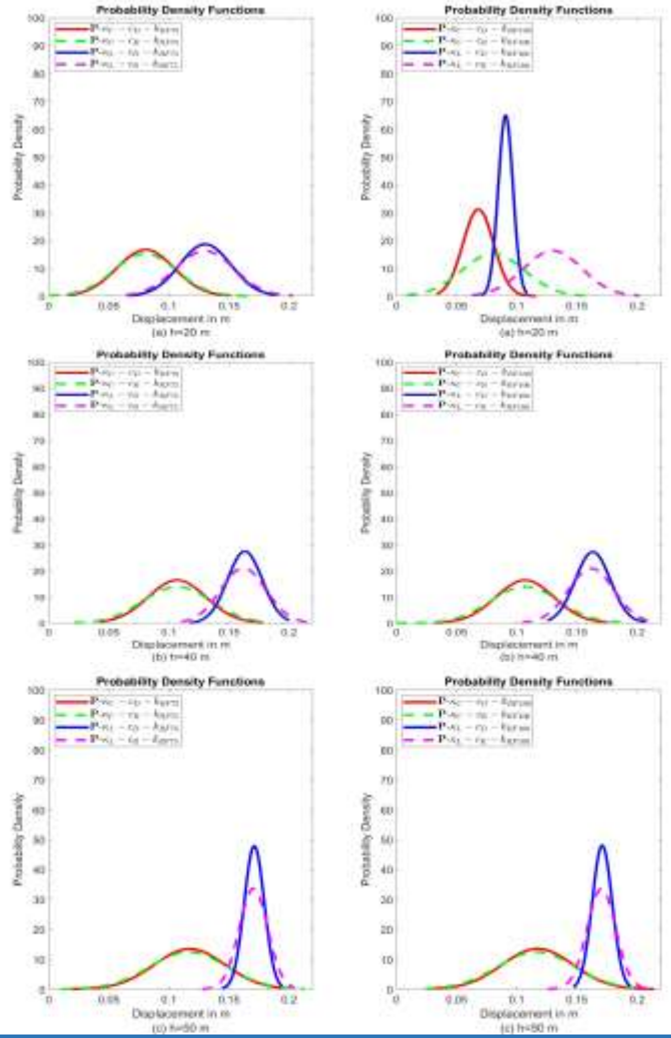
h=40 m	$k_{RF}75$-q_{fail}				$k_{RF}100$-q_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	489,56	490,88	377,69	372,69	489,88	491,13	377,50	372,69
Stdev	43,93	63,98	5,01	31,04	44,16	64,29	5,10	31,10
CoV	0,0897	0,1303	0,0133	0,0833	0,0901	0,1309	0,0135	0,0834
MAX	559	676	385	435	559	677	384	436
MIN	400	400	366	309	399	400	366	309
$\frac{MAX}{MIN}$	1,40	1,69	1,05	1,41	1,40	1,69	1,05	1,41

h=50 m	$k_{RF}75$-q_{fail}				$k_{RF}100$-q_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	459,88	452,56	358,87	357,43	460,25	452,85	358,87	357,35
Stdev	38,93	57,28	3,94	20,25	39,48	57,79	4,21	20,12
CoV	0,0847	0,1266	0,0110	0,0567	0,0858	0,1276	0,0117	0,0563
MAX	530	572	370	387	533	573	370	386
MIN	400	377	354	312	399	376	355	313
$\frac{MAX}{MIN}$	1,33	1,52	1,04	1,24	1,34	1,52	1,04	1,23

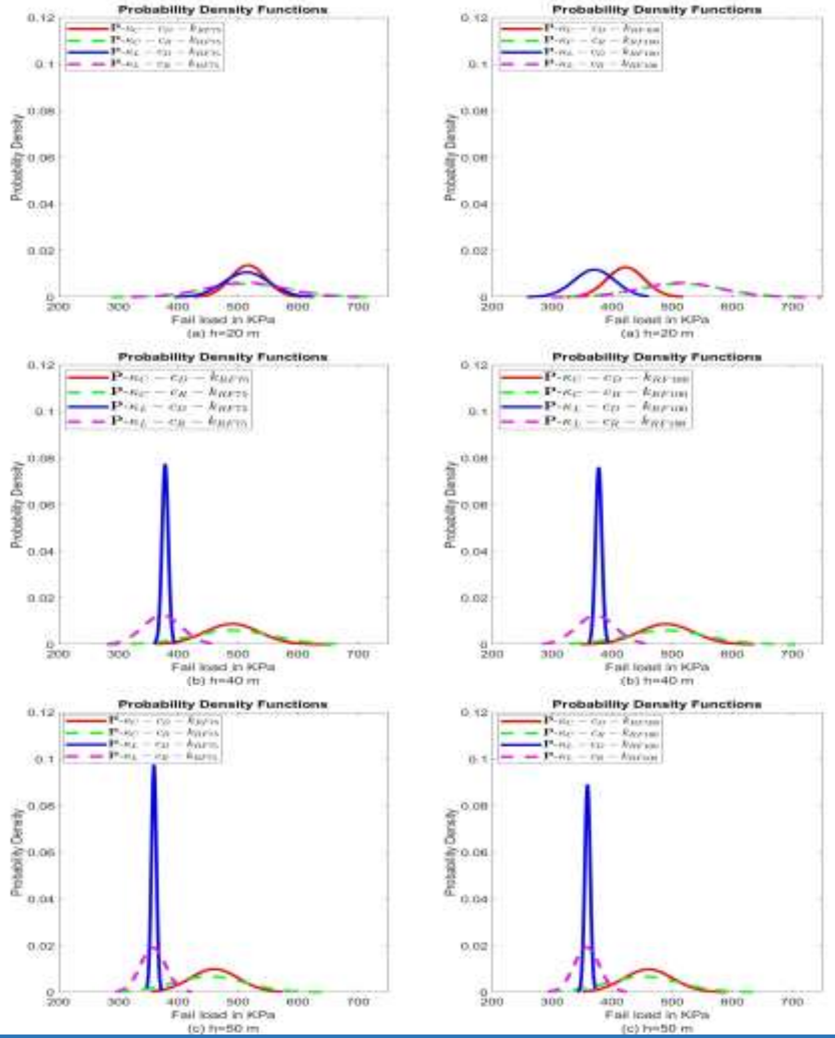
h=20 m	$k_{RF}75$-u_{fail}				$k_{RF}100$-u_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	0,0805	0,0804	0,1302	0,1299	0,0680	0,0803	0,0907	0,1299
Stdev	0,0231	0,0253	0,0207	0,0237	0,0124	0,0254	0,0059	0,0236
CoV	0,2868	0,3152	0,1587	0,1822	0,1819	0,3160	0,0656	0,1814
MAX	0,1318	0,1381	0,1641	0,1747	0,0877	0,1383	0,1070	0,1743
MIN	0,0395	0,0406	0,0829	0,0842	0,0374	0,0406	0,0848	0,0842
$\frac{MAX}{MIN}$	3,33	3,40	1,98	2,07	2,35	3,40	1,26	2,07

h=40 m	$k_{RF}75$-u_{fail}				$k_{RF}100$-u_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	0,1068	0,1068	0,1633	0,1620	0,1068	0,1067	0,1632	0,1620
Stdev	0,0234	0,0276	0,0140	0,0183	0,0234	0,0276	0,0141	0,0184
CoV	0,2195	0,2588	0,0859	0,1132	0,2195	0,2585	0,0866	0,1138
MAX	0,1500	0,1619	0,1829	0,1952	0,1500	0,1616	0,1828	0,1952
MIN	0,0515	0,0530	0,1286	0,1329	0,0515	0,0530	0,1283	0,1325
$\frac{MAX}{MIN}$	2,91	3,06	1,42	1,47	2,91	3,05	1,42	1,47

h=50 m	$k_{RF}75$-u_{fail}				$k_{RF}100$-u_{fail}			
P–	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R	κ_C-C_D	κ_C-C_R	κ_L-C_D	κ_L-C_R
Mean	0,1173	0,1163	0,1711	0,1704	0,1173	0,1163	0,1710	0,1704
Stdev	0,0286	0,0311	0,0081	0,0115	0,0286	0,0310	0,0080	0,0115
CoV	0,2442	0,2673	0,0472	0,0674	0,2434	0,2667	0,0470	0,0675
MAX	0,1687	0,1746	0,1847	0,1913	0,1683	0,1742	0,1846	0,1907
MIN	0,0611	0,0626	0,1516	0,1522	0,0611	0,0626	0,1518	0,1523
$\frac{MAX}{MIN}$	2,76	2,79	1,22	1,26	2,75	2,78	1,22	1,25



Failure displacements (m)



Failure load (Kpa)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

- The CV of failure loads is less dependent from the change of the depth than the corresponding failure displacements.
- For the same depth and material assumption in porous analyses larger output variability in both monitored variables compared to the solid analyses
- Maximum CV of failure load in porous analyses (κc case) is 47 % smaller than the input CV and for failure displacements 26 % greater than the input CV, whilst for linear distribution for κ the CV of the output is negligible in all cases.
- Porous analyses➔ Important variability reduction for failure load while for failure displacements in the case of the constant distribution for the compressibility factor significant variability increase occurs.
- Justification ➔ Bulk modulus K_b is a function of mean stress (Poroelasticity). So K_b , in porous analyses is expected with smaller values and smaller uncertainrty. Similar conclusions can be made for the failure displacements since there is no tensile strength of the soil point and consequently there is smaller surface of the BSE leading to the aforementioned results

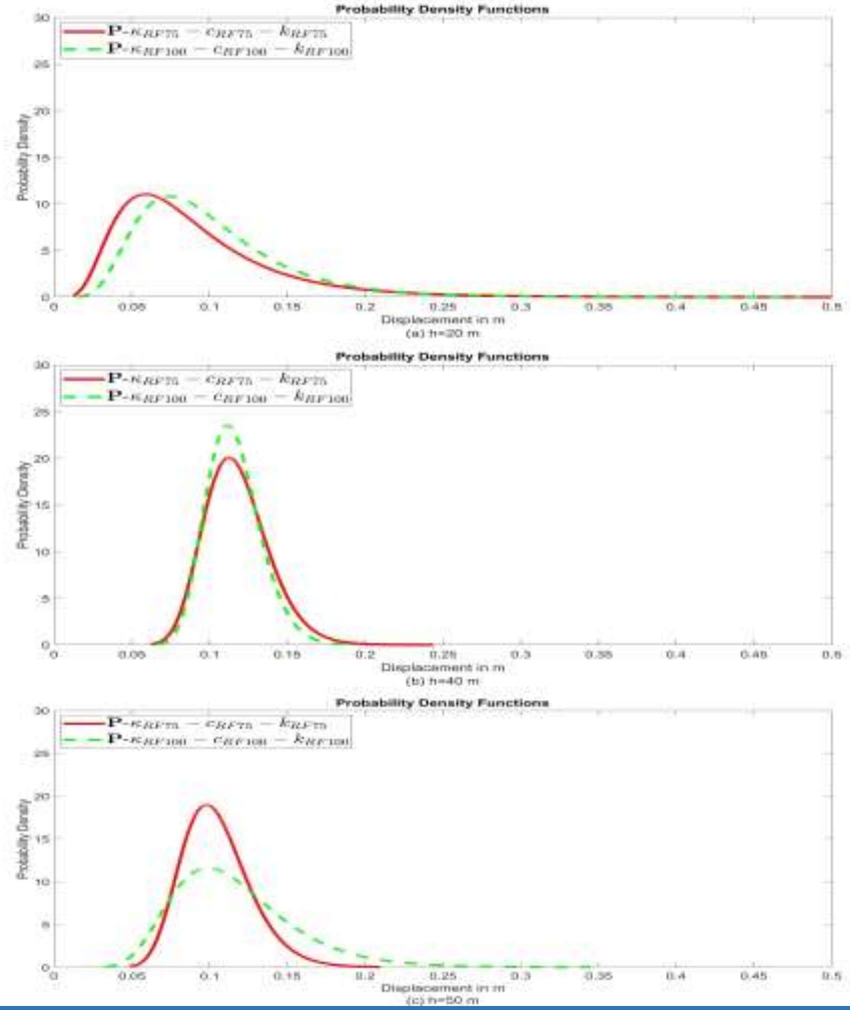
Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Porous analyses with random field representation for all stochastic material variables

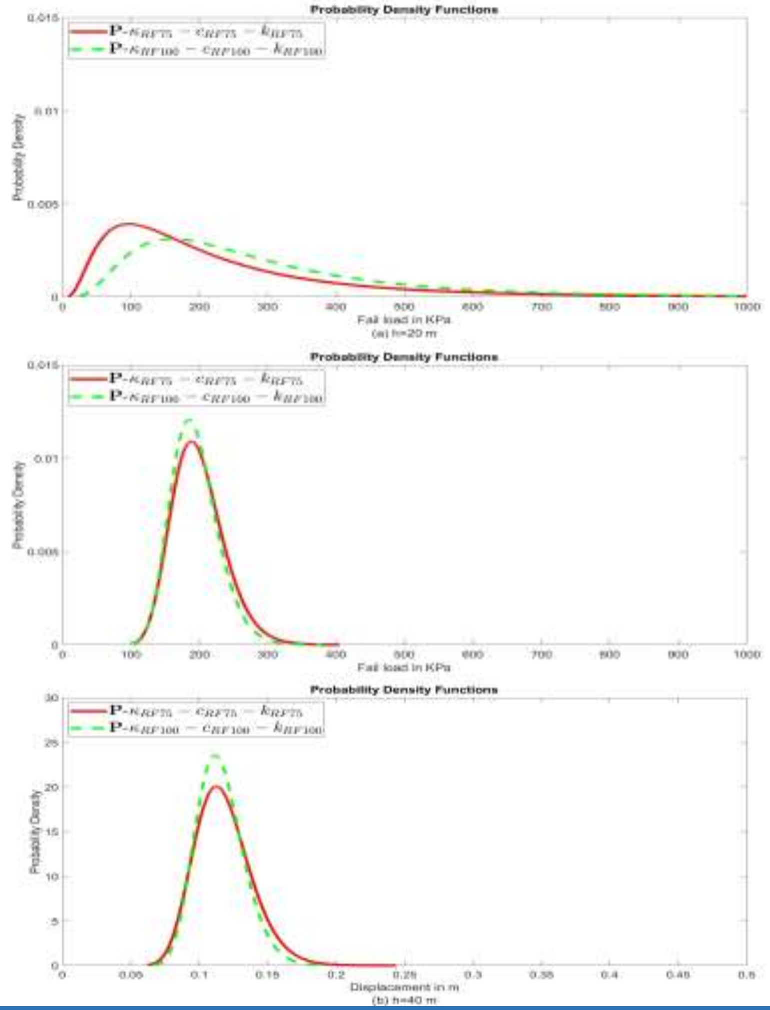
h=20 m	κ_{RF}-C_{RF}-$k_{RF}b$, q_{fail}		κ_{RF}-C_{RF}-$k_{RF}b$, u_{fail}	
P−	b=75 m	b=100 m	b=75 m	b=100 m
Mean	239,92	301,04	0,0901	0,1006
Stdev	211,57	210,63	0,0502	0,0459
CoV	0,8819	0,6997	0,5566	0,4565
MAX	600	583	0,1716	0,1650
MIN	9	22	0,0166	0,0174
$\frac{MAX}{MIN}$	68,06	26,94	10,35	9,48

h=40 m	κ_{RF}-C_{RF}-$k_{RF}b$, q_{fail}		κ_{RF}-C_{RF}-$k_{RF}b$, u_{fail}	
P−	b=75 m	b=100 m	b=75 m	b=100 m
Mean	198,88	193,56	0,1176	0,1154
Stdev	37,09	33,26	0,0200	0,0169
CoV	0,1865	0,1718	0,1701	0,1464
MAX	261	252	0,1506	0,1456
MIN	129	145	0,0777	0,0935
$\frac{MAX}{MIN}$	2,02	1,74	1,94	1,56

h=50 m	κ_{RF}-C_{RF}-$k_{RF}b$, q_{fail}		κ_{RF}-C_{RF}-$k_{RF}b$, u_{fail}	
P−	b=75 m	b=100 m	b=75 m	b=100 m
Mean	171,31	194,50	0,1046	0,1165
Stdev	40,10	67,72	0,0215	0,0382
CoV	0,2341	0,3482	0,2056	0,3279
MAX	232	285	0,1383	0,1605
MIN	72	7	0,0501	0,0045
$\frac{MAX}{MIN}$	3,22	40,71	2,76	35,80



Failure displacements (m)



Failure load (Kpa)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

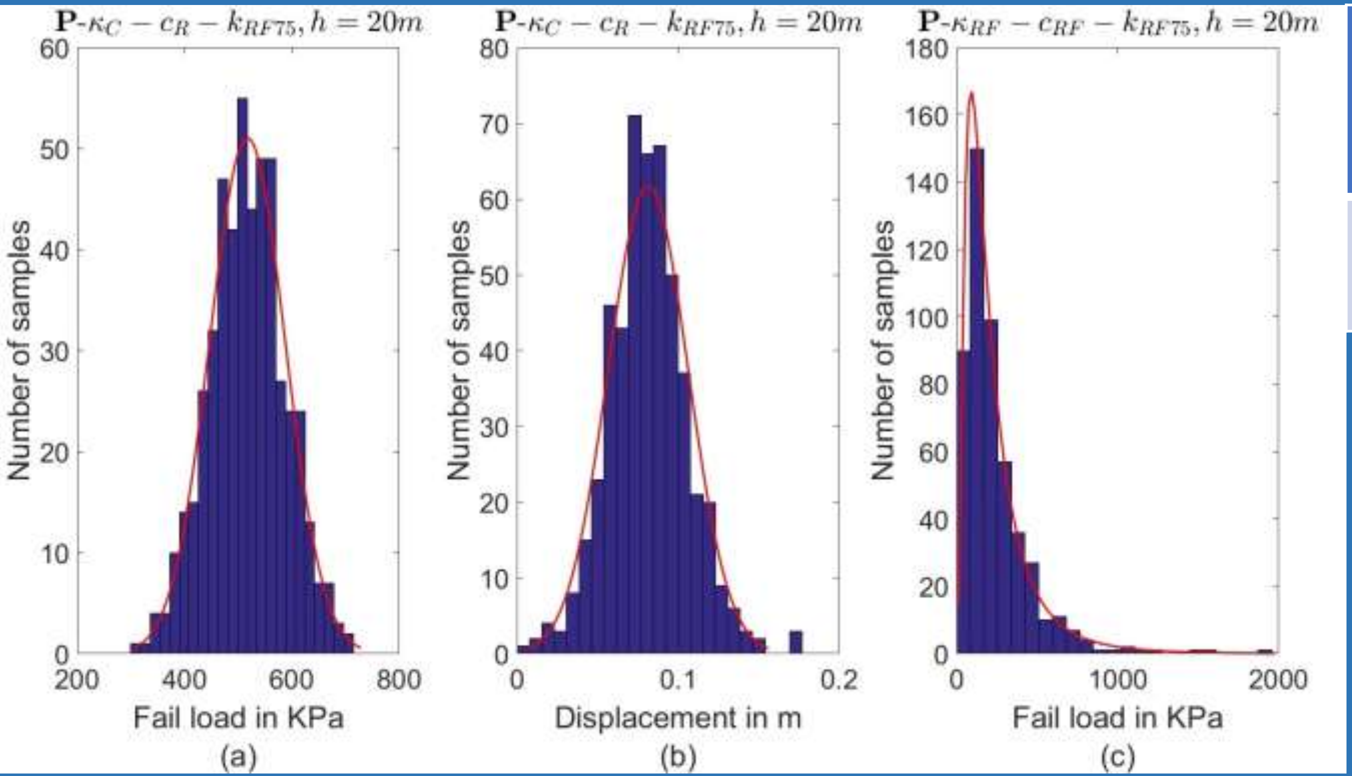
- Maximum CV of the output for failure load is 3,5 times larger than the uncertainty of the input while for failure displacements is 2,2 higher than the variability of the input.
- Mean values of failure load are significantly smaller than the porous analyses with deterministic shape functions for κ and c while for failure displacements when the constant distribution for κ is assumed larger mean values are expected in comparison with the porous analyses with deterministic shape functions for κ and c
- In the case of porous random field analyses the increase of the correlation length for 20 and 40 m reduces the CV of the output whilst for 50 m this reverses. The exact opposite phenomenon occurs for the mean values.
- Justification ➔ The integration point failure may be «from the wet side» (from the left side of the vertical halfaxis of the ellipse) or «from the dry side» (from the right side of the vertical halfaxis of the ellipse) consequently a large change of the value of c may incorporate very large deviation of the stress point of failure leading to the aforementioned results.

Numerical tests on stochastic consolidation with random linear and non linear material-Analysis of the results (Kolmogorov Smirnov Test)

- Assumption ➔ The output displacement follows the truncated normal distribution.
- Justification from Histograms ➔ Graphically this holds
- Justification from numerical test ➔ Kolmogorov Smirnov test for a sample following a distribution.
- The largest absolute difference of the theoretical and the numerical CDF is compared to the critical value. Since it is less than the critical value the null hypothesis holds and the sample follows the truncated normal distribution. Therefore the null hypothesis at the 5% significance level is satisfied to the randomly selected analyses presented to the histograms
- Despite the material non linearity the output displacement still has the Gaussian nature of the randomness

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure load and displacements

Kolmogorov-Smirnov Test



Largest Absolute Difference	**Figure (a)**	**Figure (b)**	**Figure (c)**	**Critical**
Significance level 5%	0,0752	0,0821	0,1023	0,13851

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Solid analyses

- The distribution for the critical state line inclination cR provides the larger uncertainty and the smallest minimum values of failure in both hydrostatic and deviatoric components.

- The same distribution maximizes the uncertainty of the strains which are in the order of magnitude of 3-4 ‰

- In most cases the deviatoric failure occurs.

- In the majority of the cases critical Gauss point is (2,11, 2,11, 12,11)

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Porous analyses with deterministic shape functions for the material variables

- cR case provides the maximum uncertainty and smallest minimum values of failure stresses in both stress components in 40 and 50 m while for 20 m the deterministic assumption for c provides the largest uncertainty of the output.
- The correlation length of the permeability influences notably the uncertainty of the output in stresses and strains only in 20 meters depth.
- κc-cR combination gives mostly volumetric failure and κL-cD gives mostly deviatoric failure
- Critical Gauss points are (2,11 , 2,11 , 12,11) for 20 meters, (2,11 , 2,11 , 32,11) for 40 meters and (2,11 , 2,11 , 2,11) for 50 meters and linear distribution for κ while (2,11 , 2,11 , 42,11) is the critical point for 50 meters and constant distribution for κ

Numerical tests on stochastic consolidation with random linear and non linear material properties-Failure mechanism Porous Random Field analyses

- In general, correlation length 75 m provides the maximum uncertainty at stresses.
- For the strains the critical correlation length for the volumetric component is 75 m while for deviatoric part is 100 m
- In 20 and 50 m volumetric failure is critical while for 40 m deviatoric failure is critical
- For 20 m depth and 75 m correlation length the point (2,11 , 2,11 , 2,11) is the critical whilst for 100 m correlation length is the (2,11 , 2,11 , 12,11). For 50 m and all correlation length the critical integration point is (2,11 , 2,11 , 32,11). Finally, in depth 40 meters many equally probable points may be the onset of the Meyerhoff spline.

Conclusions

- Failure load, failure displacements and failure spline and corresponding stresses-strains follow the Gaussian distribution despite the excessive material non linearity
- The compressibility factor κ plays the most important role especially when it has a Karhunen Loève distribution. Same applies for the plasticity variable c. Permeability influences to a lesser extent the uncertainty of the output.
- The amplification of the uncertainty varies from 30% to 3,5 times.
- The random field processes maximize the variability of the output to stresses and strains at failure.
- For random field processes at 20 and 50 meters the volumetric failure is critical while for 40 meters the deviatoric failure is critical.
- In the majority of the cases the integration point (2,11 , 2,11 , 12,11) may be considered the onset of the failure spline.

Thank you all for your attention.
May you stay safe from the pandemic and soon enough to be able to conference live .
Questions?